How To Learn Stuff

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Contents

Prerequisites

I help people learn Maths and Physics. This document is aimed primarily at my students, so that the examples in it are from these subjects. To follow the examples, you will need to know

- Pythagoras' Theorem;
- Very basic trigonometry;
- The essentials of integration.

If you don't know any of this stuff, don't worry. Just skip the examples!

Sources

This document is based on two main sources: one is a book by Saundra McGuire (McGuire (2015)); and the other is a book (and associated *Coursera* course called "Learning How To Learn") by Barbara Oakley (Oakley (2014)). Quite a bit of my text is shameless copy-and-paste of sections of these books.

If you are at all interested in learning how to learn, get these books. You won't regret it. There are so many ideas in these books that I don't cover here, because I want this to be a basic guide. So if you want to be an expert learner - well, you know what to do!

Any mistakes in *this* document, however, will be mine.

Notes

I prefer to put little explanatory notes for things in footnotes[1] so that they don't distract from the current moment, but you don't have to go far to find them. I hope that's OK with you.

The alternative is an endnote[1], but that disrupts the flow in my opinion. You can find endnotes at the end of the document, just before the references.

Aims

There are two aims of this document.

First I want to introduce you to evidence for practices that improve learning that I have recently stumbled upon. That's the essence of "Part I".

The second half of the document (cunningly called "Part II") is devoted to answering the question: "So, *how* can I use the theory to improve the way that I learn stuff?". It shows you basic easy-to-master techniques for acquiring knowledge, understanding, and applying it.

Techniques that you can apply to *any* course, on *any* subject, to do as well as you possibly can on the course.

Document History

Date	Version	Comments
31 January 2017	1.0	The initial creation of the document.

[1]This is a footnote.

Part I
The Theory

1 Introduction

I've divided this document up into two parts because if you want to cut to the chase and get stuck into finding out how you can learn much more betterly, skip straight to to Part II.

But I would advise you to read through the first part whenever you can. It's an interesting read and you will get a lot out of it, I promise! Part I gives the scientific evidence for the practises that I recommend in Part II. So it's kind of important.

But before you skip to Part II, finish reading the introduction!!

1.1 You Can Sculpt Your Own Brain

In the first part of this document I'm going to look at the evidence for and against different ways of learning stuff.

And probably the most important thing to say first is that your brain can be *changed* by the thoughts you have. You can sculpt your own brain, reshaping it to do whatever you want. When you *use* neural circuits in your brain, you help to build the myelin sheath over them, as well as making many other microscopic changes (Bengtsson et al. (2005)).

Practice appears to strengthen and reinforce connections between different brain regions, creating highways between the brain's control centers and the centers that store knowledge.

So...*we can make significant changes in our brain by changing the way we think*. And the more we think, the better our brains become.

1.2 A Sneak Preview

Since one of the strategies you're going to learn about that enhances the learning process is "previewing", I'm going to give you a little sneak preview of what theories we are going to cover. Here they are:

- You can change the way your brain is wired. Learning actually re-wires your brain. So it is possible for you to make your brain work better!!
- Setting up an appropriate learning *environment* is very important.
- You learn better when you *preview* material before you study it.
- You learn something much more effectively by *teaching* it.
- There are different *Learning Levels* that you ascend as you develop and mature as a learner. Understanding these levels can help make you realise the kinds of things you will need to do to succeed on any given course.
- Your brain has two main ways of thinking: these are known as *focused* and *diffuse* modes of thinking. You need to allow your brain to work in *both* modes to learn something.
- When you read, you learn better if you *practise recall* as you read, and read *actively*.
- *Regular, spaced repetition* of something is better than one long intensive session.
- *Interleaving* is good: that's when you don't work on one thing for long periods, but switch between different tasks.
- Learn how to tackle *procrastination*.
- Learn *good habits*, and get rid of bad ones.

So...if you're ready, let's go!!

2 The Learning Environment

One of the first things to think about when you embark on a period of study is your study environment. Are you planning to study on a train each day as you go to work? Do you have your own bedroom that you can use? Will you play music as you study? This sort of thing.

Ideally, you need to be able to *control* your study environment. That is, you determine what the conditions are like when you study. Sometimes that's not going to be possible. If you are in a big family and you share a bedroom, it might not always be possible to be able to study on your own. But you must do what you can to control your environment.

And what should your learning environment be like? Here is a guide:

- You should work on your own (if at all possible).
- You should not have any music, radio, TV, etc., playing as you work. When you do your exam, are you allowed to listen to music? No. Can you watch TV? No. If you always study listening to Metallica at Volume 11, you might well find it impossible to function in the silence of the exam room.
- You should eliminate distractions as much as possible. That means: put your phone on silent, non-vibration mode. Turn off the TV, radio, iPod, etc. Put a "Do Not Disturb" sign on the door. Yeah, I know how addicted to your phone you are. But just wait: when we get to Part II, you will find that I only want you to stop your phone distracting you for 40 minutes at a time. After that, you can text away as much as you want until it's time to work again. So it shouldn't be too much of a hardship.

3 Priming Your Mental Pump: Previewing

As you first begin looking at a chapter or section of a book it helps to take a "picture walk" through the chapter, glancing not only at the graphics, diagrams and photos, but also the section headings, summary and even questions at the end of the chapter. This seems counter-intuitiuve - you haven't actually read the chapter yet - but it helps prime your mental pump. So you could do that now with this document: have a look at all the pictures, and scan through the contents page. There aren't any questions at the end, unfortunately!

You'll be surprised that how spending a minute or two glancing ahead before you read in depth will help you organise your thoughts. You're creating little neural hooks to hang your thoughts on, making it easier to grasp the concepts.

There is more on previewing in Sections 15.1 and 16.1.1.

4 Teaching as Learning

4.1 Would You Study Harder to Get an A on a Test, or to Teach Material to the Class?

You might answer this question by saying things like

- "Well, I'd have to *really* know it if I have to teach it!"
- "If I'm going to teach it, I'd have to try and think of questions that I might be asked and make sure that I can answer them. I don't want to look stupid in front of the class."
- "I want to make sure everyone understands and is prepared for the test, so I need to work out how to explain the information in more than one way."

So, have a think. Thus far, have you been working in *make-an-A-on-the-test* mode, or *teach-the-material-to-the-class* mode?

Have you ever been trying to explain something you thought you understood to someone else, only to find that in the middle of your explanation there was was something you didn't grasp after all? I'm guessing the answer is yes. If you hadn't tried explaining the thing, you probably wouldn't have realised that you didn't understand it until you got into the test.

So why not try explaining things *as a means of learning* it?

4.2 You Don't Have to be a Teacher...

I've worked for quite a few years in the software industry, primarily developing the software that ran military air-traffic radar control systems. As you might imagine, a lot of it was complicated stuff.

Quite often, when I wrote a piece of code to implement an idea, the code didn't work. So I'd change the code and try again. It still wouldn't work. Most of the time I'd get it running fairly quickly, but every so often I just couldn't get it running properly.

The company that I worked for had a small room with nothing in it but a table, a chair, and sitting on the table, was a life-sized teddy bear. The idea was that when you got one of those difficult problems, what you'd do was to prepare a talk and go and explain to the teddy bear what your problem was.

And guess what? Ninety-nine times out of ten, by simply verbalising an explanation of your problem to the teddy bear, the solution would spring into your mind as you talked. It worked. Even though the teddy bear couldn't ask any questions.

Now I know that debugging code isn't quite the same as trying to learn something, but there are a lot of parallels. I was trying to teach my idea, and how I was implementing it, to the teddy bear. And as I taught, I realised something. I *learned*.

You could do this. When you are trying to learn something, it's a really good idea to get into *teach-the-material-to-the-class* mode, and then practice by trying to teach the material to your teddy bear. Or your dog. Or your younger sister. *Or your parents*.

Do you have those parents that keep getting on your back with "You're not doing enough work". "You need to work harder". "You spend too much time on your phone. Work harder!" Of course you do. Almost everyone has. Imagine the situation then *if you were begging them* to let you explain the photo-electric effect, completing the square, or pseudo-Reimannian manifolds to them every other day. I'll bet that they soon leave you alone.

So, getting your parents off your back probably justifies teaching as a means of learning all by itself. But there's a much better reason for doing it, of course. It really works as a way of learning stuff. I mean *really* learning stuff.

4.3 ...to Get Into *Teach Mode*

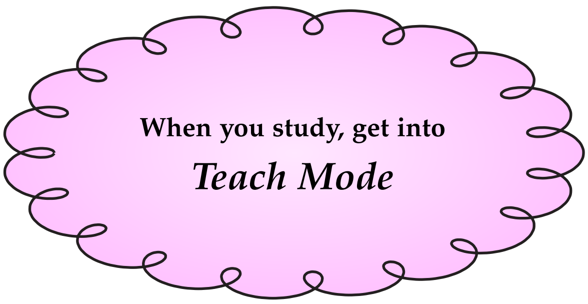
From here on, I'm going to call this thinking-how-to-learn-by-teaching-it attitude, *Teach Mode*.

4.4 Thinking About a Topic From Multiple Perspectives

Students report that teaching as a means of learning also works because they anticipate the questions that they might be asked. So they are looking at the topic from the point of view of the people they're trying to teach the stuff to. They are thinking about the possible confusions that might arise in their "students" from their explanations, as well as trying to fill their own gaps in understanding.

You will find that the moment that you start developing a deeper understanding of your subject is the moment when you start trying to teach it.

4.5 Summary



5 Learning Levels

5.1 What is the Difference Between Studying and Learning?

Some answers you might expect from this question:

- "Studying is memorizing information for an exam; learning is when I understand and can apply it."
- "Studying is short-term; learning is long-term."
- "Studying is what I do the night before a test to make an A. Learning is what I do if I know that I'm going to have to use that material later on."

One student put it this way: "Studying is focusing on the 'whats', but learning is focusing on the 'hows', 'whys' and 'what ifs'. If you focus on the 'whats', and you forget them, you can't recreate the information. But if you focus on the 'hows', 'whys' and 'what ifs', you can often work out what the 'whats' are.

So you are starting to get the idea that there might be different *levels* of learning...

5.2 The Learning Levels

Although it is a gross simplification, Figure 1 shows a hierarchy of different learning levels[2].

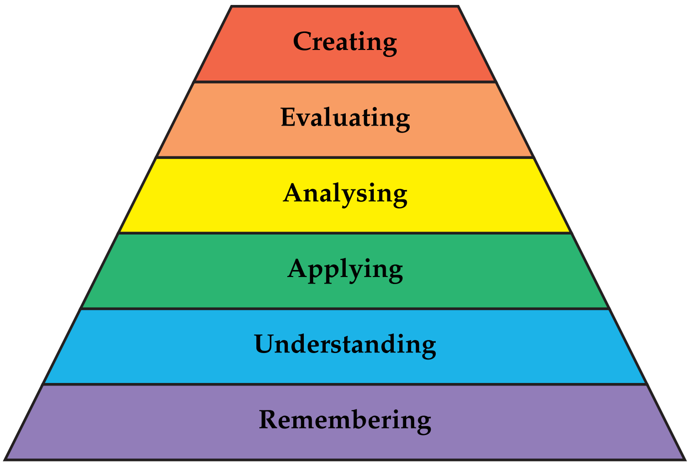


Figure 1: Learning Levels

The basic idea of this thing is that there is a difference between memorising something (*Remembering*), understanding it well enough to put it into our own words (*Understanding*), knowing it well enough to be able to apply it to different situations (*Applying*), etc, etc.

- If you are at *Remembering*, then you have memorised verbatim definitions, formulas, diagrams, etc., and you could *not* put the information into your own words.
- If you are at *Understanding*, then you can paraphrase the material. You could explain it to your teddy bear, 8-year-old nephew or 80-year-old grandmother by creating analogies and examples that apply to their lives.
- If you are at *Applying*, then you could use the information you have learned to solve problems that you have never seen before. So if I asked you to give me a mini-lecture on the identity $\sin^2(x) + \cos^2(x) \equiv 1$, you could talk to me about Pythagoras' Theorem, how they are essentially the same thing, and the other identities that are also equivalent to Pythagoras' Theorem. And you could give me examples of how you might use (*apply*) this equation to solve a problem. See Section 5.2.1.
- If you are at *Analysing*, you can take any concepts that you have learned and break them down into its component concepts[3].
- If you are at *Evaluating*, you can look at two different processes, proposed by others, and determine which is better for whatever reason (correct, desirable, efficient, ease of use).

[2]Educational professionals know this as *Bloom's Taxonomy*. The ideas for different learning levels were developed initially by Bloom, et al, in 1956 (see Bloom et al. (1956)), and were later developed by Anderson et al in 2001 (see Anderson et al. (2001)) I present here the later version, if you are interested!

[3]See how the hierarchy thing builds up? If you are at a particular level, then you can use and explain all the levels below.

- If you are at *Creating*, you can come up with your own ideas about how to solve different kinds of problems that no-one has ever told you about. Or you might come up with a different process for doing something. A different way to do something that makes the task so much easier[4].

At what level do you think you have to be at to be in *Teach Mode*?

5.2.1 An Example: Pythagoras

For example, if you are trying to learn the following trigonometrical identity:

$$\sin^2(x) + \cos^2(x) \equiv 1 \tag{1}$$

you could just learn the equation and hope you remember it when it comes to exam time. If this was your plan, then you would be at *Remembering*. But if you have a memory like mine, you are flirting with disaster adopting that strategy.

Instead, how about learning this *picture*: I don't know about you, but I find learning pictures easier than

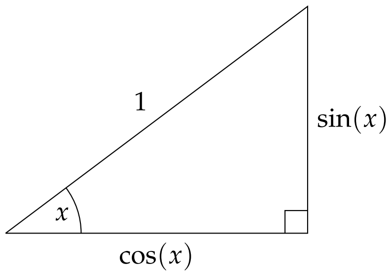


Figure 2: My Replacement for SOH CAH TOA!

learning words and equations. Now if you know this picture, then *you can work out what the equation is that you need to learn*. Let's see how we can do this. If you know Pythagoras' Theorem, and our picture is of a right-angled triangle, then you can apply the theorem to this particular triangle. That gives you the equation:

$$\sin^2(x) + \cos^2(x) \equiv 1$$

directly!

But if you understand that Equation (1) is *completely equivalent to* Pythagoras' Theorem[5], then you are certainly at least at *Understanding*. If you couldn't remember the identity, you could *work out* what the identity is from Figure 2. The picture is what you really need to remember.

So there's a difference between trying to remember facts, and understanding where those facts come from, and how they fit in to the grand scheme of things. Interesting.

And of course there's absolutely no point in remembering identities if you don't know what to do with them...

If you are interested, there's a more in-depth look at this example in Appendix A.

[4] See, for example, Smith (2012)

[5] Barbara Oakley calls this linking of ideas from one topic area to another in a subject (or even from one subject to another) "chunking". Learning a subject is all about making as many chunks as you can!

5.3 The Learning Levels Revisited

Here are the learning levels again, but with a bit of explanation of each level, together with keywords that are often associated with the different levels.

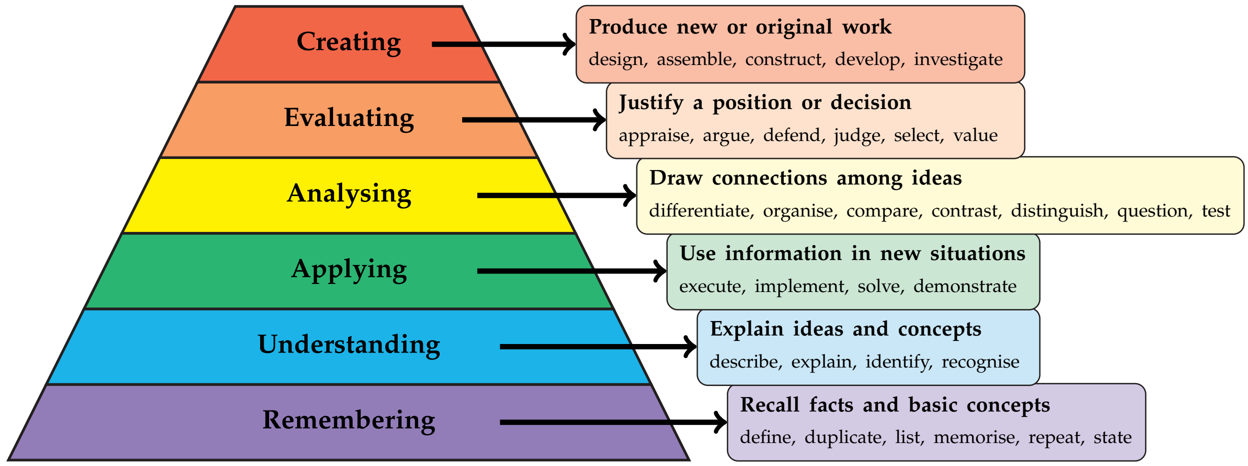


Figure 3: Learning Levels Explained

Do I need to state that the further up the hierarchy you are, the better?

5.3.1 Another Example: Goldilocks and the Three Bears

One way to get a grasp of the ideas involved in this hierarchy of learning levels, is to apply it to something very familiar.

Do you know the story of Goldilocks and the three bears? If not, go and read it *now*. The following is how the levels of learning can be applied to this story. Perhaps an interesting thing to do before you continue, would be to think for a few minutes about how *you* think the learning levels could be applied to this story. Once you've had a go, read on, and compare your thoughts with mine.

Remembering You have mastered *Remembering* when you can recall all of the things that Goldilocks used at the bears' home.

Understanding If you can give a reason why Goldilocks preferred Baby Bear's chair, bed, or porridge, then you have mastered *Understanding*.

Applying If you can reasonably predict what items Goldilocks would use when visiting another home, then you have reached the level of *Applying*.

Analysing You have mastered *Analysing* if you can think critically about the context of the story and call into question particular assumptions; for example: is it plausible that bears could eat porridge out of bowls?

Evaluating If you can give reasons why Goldilocks's behaviour could be considered justifiable by some and unconscionable by others, then you have mastered *Evaluating*.

Creating And you can demonstrate that you have scaled the entire hierarchy and mastered *Creating* if you can write your own story starring a character called Goldilocks but featuring very different themes and values. "Goldilocks and the Three Vegans", perhaps.

5.4 At What Learning Level are You Operating? At What Level Do You Need to Operate?

Here are some more questions:

- At what learning level do you think you need to be at in order to get great results at GCSE?
- At what learning level do you think you need to be at in order to get great results at A-Level?
- At what learning level do you think you need to be at in order to get great results at university?

It is my view that in order to get pretty good results in a range of GCSE subjects, you only need to be at the *Remembering* level. That might sound a bit controversial, and yes, it will vary from one subject to another, but in general, I don't think that you have to be very high up the pyramid to be praised by society as being clever at age 16.

However, when you start your A-level courses, there seems to be a massive jump in difficulty from GCSE. This step-up is particularly marked in subjects like Mathematics, Physics and Chemistry. But it is true for other subjects too. Not only is the material harder, but the questions are much harder, and you seem to be going through the material much quicker than before.

So what's the difference between GCSE and A-Level in subjects like Mathematics, Physics and Chemistry? Well, could it be that in order to do well at A-level, *it is expected* (by exam boards, examiners, and consequently teachers) that students need to be working at higher learning levels than was necessary for GCSE?

Suddenly, learning strategies that worked fine at GCSE level (don't do any work, cram the night before to remember everything you need for the exam, etc., etc.) aren't getting very good results any more. It used to hit a lot of people as soon as they took their first AS exam in the January of Year 12[6]. But at the time of writing there aren't any exams in the January of the AS year any more, the realisation that the sort of work you are doing isn't sufficient for A-level might not hit you until it's too late.

In my view, students need to be *at least* at the *Applying* learning level at A-level. And I think that it's that two-step jump in learning level that causes people grief when they make the transition from GCSEs to A-levels. At A-level, *much more is expected of you*.

So I guess the next question is: if you're at the *Remembering* level, how do you get up to the *Understanding* and *Applying* levels? And beyond? Well, that's for later. For now, just appreciate that what ever course you do, *you have to work at the appropriate learning level*.

5.5 Summary

Whatever course you do,
you have to work at the appropriate
Learning Level

[6]When such things existed. And presumably again soon, when January AS exams return, when the Minister for Education changes his mind again.

6 Focused and Diffuse Thinking

6.1 Concentration and Relaxation

Since the very beginning of the twentieth century, neuroscientists have been making profound advances in understanding the two different types of networks that the brain switches between - *highly attentive state networks*, and more relaxed *resting state networks*[7].

Here, the thinking processes involved in these two different types of networks are going to be called *focused mode*, and *diffuse mode*, respectively.

The reason I'm mentioning them is that *both focused mode and diffuse mode are highly important for learning.* See Immordino-Yang et al. (2012).

It seems that you frequently switch back and forth between these two modes in your day-to-day activities. You're either in one mode or the other - not consciously in both at the same time.

But get this: *the diffuse mode does seem to be able to work quietly in the background on something you are not actively focusing on*.

Focused mode thinking is essential for studying mathematics and science. It involves a direct approach to solving problems using rational, sequential, analytical approaches. Turn your attention to something, and *bam*: the focused mode is *on*, like the tight penetrating beam of a searchlight.

Diffuse mode thinking is also essential when studying mathematics and science. It allows you to suddenly gain an insight on a problem you've been struggling with. It is also associated with "big-picture" perspectives. Diffuse mode thinking is what happens when you relax your attention and let your mind wander. The relaxation allows different parts of your brain to hook up and deliver valuable insights on a problem.

So when you study difficult subjects, you have to let your brain work in both focused and diffuse modes to get the best out of it.

Focused mode thinking takes much more mental effort than diffuse mode thinking. So by regular switching from focused to diffuse thinking, you are also *giving your brain a rest*!!

6.2 The Einstellung Effect

Another issue about studying difficult subjects is the *Einstellung Effect*. This is where you get stuck in a mental groove. You get one idea stuck in your head about how to solve this problem, and keeping focused on that one thought prevents a better thought from surfacing (see Bilalić et al. (2008)).

This kind of thing happens a lot in mathematics and science because sometimes your initial intuition about what's happening is misleading. You have to *unlearn* something in order to learn it properly.

The Einstellung effect is a frequent stumbling block for students. It's not just that sometimes your natural inuitions need to be restrained, it's that sometimes it is rough even figuring out where to begin, as when tackling a homework problem. You bumble about, your thoughts far from the actual solution, because the deep groove of the focused mode prevents you from springing to a new place where the solution might be found. Notice: *the Einstellung effect only occurs in focused mode*.

So...if you are trying to understand or figure out something new, your best bet is to turn off your precision-focused thinking and turn on your "big picture" diffuse mode, long enough to be able to latch on to a new, more fruitful approach.[8]

[7] See Mangan (1993), Cook (2002), Raichle and Snyder (2007), Takeuchi et al. (2011), Andrews-Hanna (2012), Moussa et al. (2012), for example.

[8] The best idea I've ever had in my life came to me when I was in diffuse mode. In fact, I was asleep. I was trying to solve a particularly difficult problem while working on a military air-traffic control system. The problem was exacerbated by the fact that the customer kept changing the requirements. Four of us had been working on this problem for about six months. And the software was getting more and more complex, larger and larger, as each requirement change came in. And because when each change was received we had to re-write large chunks of code, then we were continually having to re-test the system to make sure all the previous requirements (that we'd already tested lots of times) were still satisfied.

This one magnificent night, the solution came to me in a dream. It was astonishing in its simplicity: I could replace all the original code *by only a few lines of code and a small database of rules*. The following weekend I spent the whole of Saturday and Sunday stripping out the original code (that took two days) and replacing it by my new solution (that took about 20 minutes). And it worked. And the real beauty of the new solution: when a new requirement came in, *the code did not have to change*: I just changed the rules in the database! And testing: because the code didn't change, we only had to test for the changes to the rules.

7 Practising Recall When You Read

7.1 Using the Textbook

Increasingly often I am coming across students who haven't been given a textbook when they start a course. Whatever the reason, financial constraints on the school, the teacher not believing it necessary, whatever, *it is necessary*. It is vital. And there are many reasons for this. Such as...

- How can you preview material before a class if you don't have anything to look at?
- The textbook will say the same thing, but *in a different way*, to the teacher. That's good. If the teacher says something that you don't understand, then maybe getting someone else (the author of the book) to go through the same ideas in a different way will make it clear. Or clear*er*. So ideally you want several textbooks. For any given idea, one of them surely will be able to explain it in a way that you can understand.
- There will be lots of examples for you to work through.
- There will be lots of questions to test your knowledge and skills.
- However good the teacher, their notes will be insufficient. They know the material inside out (hopefully), but when they teach something, they miss lots of things out that are obvious to them, but not to the students. It is impossible to avoid this. I do it myself. It still happens in books, of course, but it's much worse for someone standing in front of a class because of the pressure of time to get through the stuff, and the way the teacher is going through the material is perfectly logical to him. So to fill in the gaps, students need extra sources of material to use to perfect the notes they take[9]. If they don't, they'll fill in the gaps themselves...with potentially dire consequences...

And another thing: know how to use a textbook! I encounter the following scenario with students all the time. We're sitting together. We're going through a question that has the word "power" (or whatever) in it. I ask: "What is power?". Blank look. "How can you find out what power is?" Blank look. I stare down at the textbook. No reaction. "Could it be defined in your textbook?" The student starts looking though the book by seemingly flicking through pages randomly. They don't know that there is a "Contents" page, or how to use the "Index"!!

8 Regular, Spaced Repetition

Research has shown that if you try to glue things into your memory by repeating something twenty times in one evening, for example, it won't stick nearly as well as it will if you practice it the same number of times over several days or weeks (Baddleley et al. (2009), Carpenter et al. (2012)).

Flash Cards are really good for this. Say you want to remember the definition of, symbol for, and units of the physics concept of *Power*. You could take an index card and write (*writing* seems to help you more deeply encode information, so making your own flash cards is a really good idea)

Power

on one side, and

$$\text{Power} = \frac{Energy}{Time}$$
$$\text{or } \; P = \frac{E}{t}$$
$$\text{(The units of power are Js}^{-1}\text{, or Watts W)}$$

So, you look at the side with the *Power* on it, and see if you can remember what's written on the other side. If you can't, flip it over and remind yourself of what you are supposed to know. And when you do this, *read the information out loud*. If you can remember what you need to know about *Power*, put the card aside.

[9]When I was at university and it came to start revising for the final exams, I looked at my notes and I couldn't make any sense of them. I'd obviously missed lots of things out during the lectures because I couldn't write fast enough (and because they weren't in the lecture!). Panic set in. How can I revise if I can't understand my own notes? So I borrowed other peoples' notes. They'd missed stuff out too, but once I'd borrowed several sets of notes, I was able to piece together what happened in the lecture, and supplement my notes so that I could read them, understand them and revise from them. It is only when you understand something can you revise it. And I had to do this for all the exams I was sitting. It took a while. But it was certainly not wasted time, however, because the action of trying to understand and supplement my notes gave me a great start in learning it. In fact, by the time I had struggled through this process of supplementing my notes to a point that I could understand them, I was pretty ready for the exams.

Then go and do something else for a while. How long that while is could vary from minutes to days, depending on how well you recalled the information about power. The better you remember the stuff, the longer you should leave it. And then test yourself again on the *Power* card.

You will build up a stash of lots of cards. You could group them by subject and topic. Then, one of the ways of learning about C1 Coordinate Geometry would be to take out your C1 Coordinate Geometry cards, shuffle them, and test yourself on twenty of them. Those that you could remember, put to one side. Those that you couldn't, put back into the remainder of the twenty cards and shuffle them again. That way that card will come up again soon. Continue this until you can remember all the cards.

By increasing your spacing between sessions using the C1 Coordinate Geometry cards, you will become more certain of mastery: you will lock the information firmly into place. See, for example, Dunlosky et al. (2013) and Roediger and Pyc (2012).

There are well-designed electronic flash card systems, such as Anki that have built into them the appropriate spaced repetition time to optimise the rate of learning new material. You should check Anki out. I intend to try and produce my own flash cards for A-level Maths and Physics in Anki as soon as procrastination allows.

If you have different decks, one for each subject topic, then you could practice with the C1 Coordinate Geometry cards, then practice with the Unit 1 Electricity cards, then go back to the C1 Coordinate Geometry cards. That would be an example of...

9 Interleaving

Interleaving is doing a mixture of different types of activity as you study.

For example, don't spend the whole night studying nuclear physics. Split up your study periods (see Section 13) so that you study nuclear physics for a while, then switch to integration by substitution. After that, you could go back to nuclear physics again.

So you are just mixing up different topics in one study session, like the layers in a sandwich.

Or you could interleave when you spend one study period doing your differentiation homework. Let's say that you had been tasked with doing six problems from Exercise 7C from your textbook. After doing two of the questions, go and do a question from Exercise 7B. Then do two more of the questions from your homework. Then do a question from Exercise 7A. Finally finish off doing the last two questions from your homework.

Why? Research has shown (Rohrer and Pashler (2007)) that it can be a waste of valuable learning time spending all your time on one activity. Once you have grasped an idea, *overlearning* it (continuing practice when something is understood) does not reap any additional benefits.

Instead, if you switch your brain to think about something else for a while, and then come back to your original work, then you will start to learn about *when* to apply a particular technique, as well as *how* to apply it. Because you know that Exercise 7C is testing your ability to differentiate using the chain rule, you know that you need to apply the chain rule to every question. But by looking at other differentiation questions for a while, then coming back to Exercise 7C you will be learning under what circumstances it will be necessary to use the chain rule.

And that's really important for the exam, because they don't ask you questions like "Differentiate ... using the chain rule". They ask questions like "Differentiate ...". They don't tell you which technique to use. It's up to you to learn *when* a technique should be used, as well as *how* to use it.

Interleaving helps you do just that.

10 Regular Testing

There is powerful evidence demonstrating the effectiveness of testing as a way to reinforce, deepen and enrich learning. In articles for the *New York Times*, science writer Benedict Carey (Carey (2010), and Carey (2014)) shares evidence from Roediger and Karpicke (2006) and Pennebaker et al. (2013) that illustrates the power of testing.

Practice testing works. If you want to do well on a test or on an exam, you need to practice first. It may be obvious to you that if you want to study to take an actual exam, then you need to do some past exam papers as practice. But for the same reason, it's good to practice for tests!

11 Procrastination is the Pride of Princes

As my school Latin teacher used to say.

11.1 The Problem

This is the last section of this document that I wrote. Perhaps I've been putting it off.

Procrastination is one of the biggest problems that students today have to face. "Before I start my homework, I'll just check Facebook, Twitter, Tumblr, Instagram and email..." Two hours have gone. And even after homework has started, the ping of the arrival of a new text provokes the instinctive reaction to stop what you are doing and check the text. As if it is the most important thing in the world.

As a student in the modern world, you must develop strategies to prevent distractions and procrastination.

So how do you do that? Well, you can start by trying to understand how procrastination works. And the first thing to understand is that we procrastinate about things that make us feel uncomfortable (Ainslie and Haslam (1992), Steel (2007)). If you are a mathphobe, you can actually feel *pain* by thinking about doing mathematics (Lyons and Beilock (2012)).

But get this: it's the *anticipation* that's painful. When the mathphobes actually *did* the mathematics, *the pain went away*. The dread of doing a task takes up more time and energy than actually doing the task.

Now I'm all for avoiding pain. But the pain is in the *dread*, not in the *task*! If we let the dread win, so that we avoid doing the task, then that's great in the short term, but it's disastrous in the long-term. It's like taking small amounts of poison every day. In the short-term it's not so bad, but there may not be a long-term...

Procrastination is like an addiction. And it works like other addictions: it works on *cues*. You have a maths problem to solve. *Ping*. You lapse into your comfortable procrastination response: read the text. Feels good. You really should get down to that essay. *Ping*...

11.2 The Solution

Barbara Oakley calls bad habits "zombies". And when you have fallen into a bad habit, you are in "zombie mode". So...how do you prevent yourself from getting into zombie mode?

To understand how, we need to look a bit deeper into zombies. Here's how they work:

- **The Cue**: this is the trigger that launches you into zombie mode. The *ping* of a text arrival, for example.
- **The Response**: this is your zombie mode. It's what you do following the cue. Read the text, obviously.
- **The Reward**: Habits reward us by giving us small dollops of pleasure. Procrastination is an easy habit to develop because the reward (moving your mind to something more pleasant) happens so quickly. But good habits can also be rewarded. Findings ways to reward good study habits is vital to escaping procrastination.
- **The Belief**: habits have power because of your belief in them. For example, you might feel that you'll never be able to change your habit of leaving things until the last moment. To change the habit, you'll need to change your underlying belief.

The trick to changing your behaviour is to look at your reaction to a cue. *The only place you need to apply willpower is to change your reaction to the cue*.

So the secret to eliminating procrastination is to:

- **The Cue**: Look for the cues in your life. You need to identify *as many of these as you can*!
- **The Response**: Prevent the cues from affecting you. If you are a slave to your phone, then when you want to study, turn it off. "Whoa! Turn off my phone? No way, man!" But...as you will see in Part II, you will only need to do this for a short amount of time.

- **The Reward**: Reward yourself for the work you do. If you like films, well, watch one. The thing is to develop a routine that includes rewards. And you will find that a feeling of accomplishment for completing a task is quite often its own reward. Coffee. Texting. Web surfing. Start to think of these things as rewards for activity.
- **The Belief**: Start believing that your new system will work (it will!) and that it will see you safely through your exams (it will do that too!). A powerful tool here is *mental contrasting*: compare where you are now with where you want to be. *Because of this system, you will achieve what you want to achieve!*

And you can start putting this into practise by focusing on *process* rather than *product*. That means concentrating initially on just getting into the routine, without worrying too much about what you actually accomplish in your routine.

In Part II we will discover that the best way to work is to do little bits of work, separated by breaks. I call this little bit of work (with no distractions) followed by a break a *sprint*. So when you start working in sprints, just work. Don't get hung up on what you are achieving. Achievement will come. Just get into the new routine of a bit of work, followed by a reward. Bit of work followed by a reward.

Barbara Oakley devotes a lot of her book to tackling procrastination. After reading this document, and you still have procrastination issues, then I would *strongly* advise you to get her book, and look at the ideas that she and a lot of her students have come up with for getting to grips with this pernicious devil.

12 Habits: Good and Not So Good

This is a kind of summary of the whole of Part I.

12.1 Top Ten Good Habits

Use recall After you read a paragraph, look away, and recall the main ideas. *Don't highlight*. Try recalling ideas as you walk to the bus stop, or when you are in a different room from the one where you originally learned the stuff. The ability to recall is one of the key indicators of good learning.

Test yourself On everything. All the time. Flash cards are your friend.

Get into *Teach Mode* Learn things as if you had to give a presentation on Friday. In fact, why don't you give a presentation on Friday? You could give it to your Mum, or the dog.

Space your repetition Spread out your learning of a given topic and do a little every day.

Do a mixture of activities as you practice Never spend too long doing one particular kind of thing, such as solving problems, using flash cards, reading, writing a report. Mix it up and work on different kinds of activities. Do a little of each.

Alternate different problem-solving techniques as you practice Even when you are doing homework questions, mix up the kind of questions you are doing, so that you learn *when* to use a technique as well as *how* to use a technique.

Take breaks If you get stuck on something, take a break. Do something else for a bit. Come back to it later, and quite often you get a different insight into your problem.

Get into focused mode When you study, split your study time into short lumps of time when you can fully concentrate on something. Eliminate distractions during this time. Then...take a break.

Eat your frogs first Start working on the *hardest* things first. If you get stuck, switch to something else. Then come back to the hard stuff, ..., switch, ...

Make a mental contrast Imagine where you have come from, and constrast that with where your studies will take you. Remember your dream!

12.2 Top Ten Bad Habits

Waiting until the last minute to study If you were preparing to run a marathon, would you only start training sessions the night before? Procrastination is the single, monumentally important, keystone bad habit.

Reading passively Sitting passively, and letting your eyes run over the page. Unless you can prove that you can recall the information in the book, reading is a complete waste of time.

Highlighting This is a really good technique to fool yourself into thinking that you know something. But actually, all you're doing is moving your hand.

Glancing at a problem solution and thinking you know how to do it This is one of the worst errors students make when they study. You need to be able to solve a problem step by step, *without* the solution in front of you.

Repeatedly solving problems of the same type that you already know how to solve If you were a footballer, and you're really good at heading, would you prepare for the final this coming Saturday by only practising your heading?

Letting study sessions with friends turn into chat sessions If you work in groups occasionally (a very good thing!), then make sure that the members of the group focus on the work before enjoying the fun.

Neglecting to use the textbook before you start working on problems Would you dive into a pool before you learn how to swim?[10]

Not clearing up things you don't understand There are many ways of doing this: your teachers, tutors, peers, books, YouTube, the internet in general...

Thinking that you can learn deeply while you are being constantly distracted Every tiny pull toward a text, an email, a conversation means that you have less brain power to devote to learning. So it won't happen as well as it could.

Not getting enough sleep Your brain pieces together problem-solving techniques while you sleep. It also practices and repeats whatever you put in mind just prior to falling asleep. Prolonged fatigue builds up toxins in the brain that get washed away during sleep. If you don't get a good night's sleep before a test, *nothing else you have done to prepare for the test will matter*.

[10] Actually, I did once. Well, the equivalent, anyway. Nearly drowned. Seriously.

Part II
The Practice

So...how do you go about putting all this theory into practice? You use the strategies that are presented in this section. And they fall into three main categories:

- The "Sprint",
- The "Study Cycle",
- Preparing for tests and exams.

Preparing for tests and exams seems a pretty obvious thing you might want to do on a course. And you're right. I've covered the topics of *revision* and *exam technique* in another document (Smith (2015)).

But what is a "sprint", and what's the "Study Cycle", then? Aha! I'm glad you asked!

13 The Fundamental Unit of Work: the *Sprint*

When you are working on some aspect of your course, reading the textbook, answering homework questions, etc., then you can either work on your own, or with others. Working in groups is dealt with later. For now, let's think about the way that you work most of the time: alone.

I want to describe here the main way that you will work on your own. Since all big jobs can be broken down into smaller pieces, and those smaller pieces can be broken down into even smaller pieces, etc. etc., I want to introduce you to the fundamental unit of your work: I call it the *Sprint*[11].

13.1 The Basic *Sprint*

What you do is to break up all your work on a task into *sprints*. Each *sprint* can be as short as 20 minutes, or as long as 90 minutes, but I recommend that you start out using 60 minutes as a guide. And this is what you will be doing in those 60 minutes:

- Do an active learning task ($\approx$ 40 minutes). I'll talk more about precisely what you do in these 40 minutes later!
- Take a break! ($\approx$ 20 minutes). Do anything you want. Go for a walk. Call one of your friends. Watch TV. Use social media. Anything you want. This part of the process is vital! It restores energy and motivation, gives you a reward, and allows what you have been doing for the last 40 minutes to "sink in"[12].

Once one *sprint* is complete, you can start another one. And another.

Sprints use the theory of focused and diffuse thinking patterns (Section 6). During the $\approx$ 40 minutes of activity, you will be in focused thinking mode. During the $\approx$ 20 minutes of break, you will be in diffuse thinking mode. Both modes are essential for your brain to assimilate and process new information, and to practice tasks.

If you are new to these ideas, start organising your work into basic *sprints* from *now*. Later, when you get more experienced, come back and read the next section on more advanced *sprint*ing!

So when you are working on your own, split your time into sessions of approximately an hour each. And remember: in that hour, you're only working for about two-thirds of the time.

13.2 The Advanced *Sprint*

When you get really good at doing *sprints*, you can add a couple of extra bits.

In an advanced *sprint*, each *sprint* can be as long as works best for you. For me, I like making my *sprints* an hour long. And this is what you will be doing in your advanced *sprint*:

- Set *specific* goals for the session ($\approx$ 2 minutes).

[11] This is my name. McGuire calls them "Intense Study Sessions". I don't like the sound of "intense". It smacks of hard work! And they are not really *intense*. *Intense* really just means what you do when you are concentrating on something. I've borrowed the name *sprint* from modern software development practices. I like it because the name gives you the essence of the thing: a brief spurt of activity followed by a period of rest!

[12] To see just how important this break is, see, for example, Howard (2006).

- Do an active learning task ($\approx$ 40 mins).
- Take a break ($\approx$ 20 mins).
- After the break, review the session ($\approx$ 5 mins). During this time you should assess how well you learned the material. If you've identified things that you don't understand, write them down.

Setting goals focuses the mind as to what you are trying to achieve in this *sprint*. It raises questions that you want to try and answer during the *sprint*.

Doing an assessment allows you to think about how you can make your learning practices better in the future.

13.3 Summary

When you are working on your own, split up your effort into ***Sprints***

14 Working in Pairs or Groups

Working in pairs or groups can be a powerful supplement to other learning strategies.

In groups, you will have the chance to *teach*. You also get the opportunity to accurately *judge* your own learning as compared to your peers. See Cook et al. (2013) for more details. Cook found evidence that in groups you are much more likely to experience and assess how other people think, and consequently you will think more about how you yourself think. And this has great benefits.

Obviously, to be effective, group work should be more than just socialising. To get around this problem, you could look at, for example, the "Study Group Starter Kit" produced by the Center for Academic Success at Louisiana State University (where Saundra McGuire works). Although this kit is somewhat over the top, it does try and implement ideas from such research as Vygotsky (1980) and Bruner (1985), which essentially say that when doing group work, focus on the work first, and while working, it's best to do both discussion and problem-solving activities. Pretty obvious, really.

15 The Study Cycle

The study cycle is the daily routine that you get into when you take a course. This is what will happen most of the time. It's the old familiar go-to-classes, do-the-homework sort of thing.

But of course, we are trying to improve the way that we think about how we learn. And there are things that we can do to improve this daily routine. You may have never given any thought to how we can improve the process of assimilating material. Experts have. One thing they have come up with to help is the "study cycle". So, to get the most out of going to classes, you should ideally do the following five things as part of your daily routine:

- Previewing material *before* it is taught to you
- Go to classes
- Review the material
- Work on the material
- Assess your learning

And in the next few sub-sections, we will talk a bit more about each of these things. Except that is, for "work on the material", which is covered in Section 16.

15.1 Previewing Material

This step is all about previewing material *before* you come across it in class.

For this to be possible, you will obviously need to know what you will be doing in forthcoming classes. That will be relatively easy at university, where course timetables often detail lecture contents[13], but at A-level, it might not be so clear what will be taught in next week's lessons. So what do you think the answer to that problem might be? How about *get your teacher to tell you what each of the next few lessons will be about*. This ought to be a regular feature of each week's timetable.

So, let's assume that you know what Wednesday's Maths lesson will be about. It's going to be about solving quadratic equations. Now what?

There should be a textbook that is being used to support your course. If you haven't got your own personal copy of it, get one. Once you know that Wednesday's Maths class will be on solving quadratic equations, you can look up the relevant section(s) in the textbook, and skim through them. Rocket science.

How long do you need to spend doing this? Obviously that will depend. But it could be as little as 10 minutes[14]. If you're new to previewing, I should start by allocating longer than that, and I would recommend spending longer than that, but if you are pushed for time, just do what you can. *But do it*. For *every* class!!

So what *exactly* are you trying to achieve by previewing the material? Well, the idea is described fully in Section 16.1.1 where it is applied to reading material. But you do the same thing when you are previewing material before class as you would when you are previewing material you are about to read carefully.

15.2 Attending Classes

Again, this is more appropriate to university students than to A-level students, but the simple idea is clear: *go to all your classes*. Or at least, as many as your health and other extraneous circumstances allow.

If you've previewed the material for a class, there are several advantages when it comes to attending the class. As you have already skimmed through the material,

- class time will be a bit like *revision*;
- there will be things that you don't understand. In that case you can write down questions. Questions that you can then ask in the class.

And when you are in class, *take notes, by hand*. There is research (see, for example, Mueller and Oppenheimer (2014)) that indicates that taking notes by hand has greater learning benefits than taking notes electronically. Reasons for this include:

- Paraphrasing is better for learning than transcribing text verbatim. Mark Twain is often credited with this famous quote: "College is a place where a professor's lecture notes go straight to the students' lecture notes, without passing through the brains of either." We must guard against this. If you think about what you are writing as you write it, learning may well occur! If not, ...
- What software tools do you use to take notes? What do you do if there is an important graph to copy? Or some other important and complex picture? Or a table? Mathematical equations?

If you miss a class, borrow someone else's notes (or the notes from more than one someone!) and create your own notes for the class.

15.3 Reviewing Material

As soon as possible after class, you should review the material from the class, and explain it to yourself (and others - see Section 4.2). This review helps to clarify and solidify the ideas from the class. And remember to be in *Teach Mode* when you do this.

Have you ever watched a film more than once? Of course you have. Have you ever found things from the second (and subsequent) viewings that you missed the first time? Of course you have. So the power of reviewing material is clear then.

If there are gaps in your notes, fill them as soon as possible!

[13] Not always, of course...
[14] Which could be the 10 minutes immediately prior to the start of the class, if you get there early!

15.4 Assessment

By engaging in self-evaluation, you can see how well your learning is progressing. Maybe you were trying flash cards this study session. Did they work? Maybe concept mapping would have been better? Are you having serious trouble understanding a certain topic? If so, you have identified that you need help. Get it!!

15.5 Summary

For each class of the course...

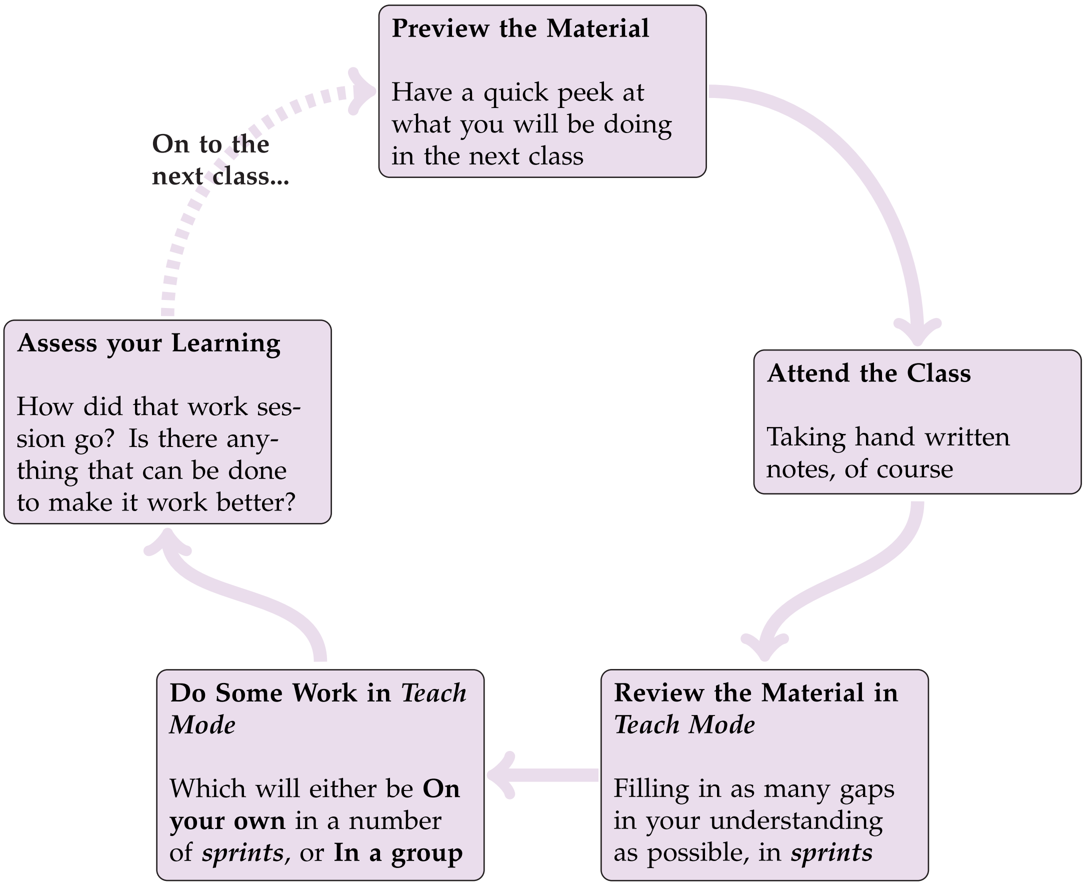


Figure 4: The Study Cycle

16 What to Do in a *Sprint*

Hang on. You've talked about *sprints* in Section 13. Why didn't you put *this* stuff in *that* section?

I'm *interleaving*! Cunning, eh?

So...what do you do when you are in the "active learning task" bit of a *sprint*, all by your own self? Well, I'll tell you. You can do one of these things:

- you can read something, or
- teach something to someone else, or
- solve problems (from homework or past papers, say), or
- create your own practice exams, or
- remember stuff using flash cards.

That's it.

Let's cover each of these in turn.

The thing you will probably do most often in a *sprint* is read stuff. If you can't read, learning is *much* harder. So I'm going to tell you how to read. "What?", I hear you cry, "I know how to read!!". Do you, indeed...

16.1 How to Read

What happens when you read? If you're like most people, you begin to read, and all is well until your mind starts to wander. But your brain doesn't immediately realise that it's not paying attention to what you're reading because your eyes are still tracking the text. It's not until you get further down the page that you realise "Oh, I stopped paying attention ages ago, and have no idea what I'm reading about".

At this point, what do you usually do? If you're like most people, you'll go back to the beginning, and start again. But this time, of course, you're determined to concentrate harder to stop yourself from coming off the rails. And...you get a bit further before you realise you are thinking about dinner as your eyes skim over the words.

And then what do you do? Start again...[15]

So just reading the thing, *passively*, isn't going to work very well. So what can we do instead? Obviously we have to be more *active* when we read. What does that mean? How can we read *actively*?

16.1.1 Preparation 1 : Previewing

[The same ideas described here apply to previewing material before going to a class. See Section 15.1.]

The first thing to do is to give yourself a *preview* of what you're about to read[16]. The brain is much better at learning when it has a big picture, and then fills in the details. See, for example, Klingner and Vaughn (1999). How do you give yourself that big picture? You can do these things (amongst others!):

- look at section headings;
- look for text in **bold** print;
- look for text in *italics*;
- look for charts, graphs and pictures, and read their captions.

If you have some idea of, or the context for, what you are about to read, your brain can recognise and process much more information than if you just dive headlong into a piece of text.

As an example, let's say that I'm tasked with reading a chapter on the basic properties of waves. So I follow the guidelines above and I notice that in the chapter on waves there are section headings called:

- "Introduction and Definitions",
- "Derivation of $v = f\lambda$",
- "The Principle of Superposition",
- "Phase Difference", and
- "The Equation of a Progressive Wave".

There are also several diagrams, with the captions

- "Displacement against distance for..."
- "Displacement against time for..."
- "To illustrate the principle of superposition", and
- "Sinusoidal wave".

Done! You get much more out of this if *you write this stuff down*. So do that.

16.1.2 Preparation 2 : Ask Questions

Just as no four-year-old likes hearing "you have to...", neither does your brain. You have to give yourself a *reason* to read.

How can you do that? One way is by coming up with questions that you want answered as you go through the "Preparation 1 : Preview" process. That taps into your innate curiosity, and motivates you to do the reading!

So what questions can I ask myself about this chapter on waves? How about these:

[15] This sort of thing happens even quicker if you're tired. So that's one of the reasons for building breaks into the *sprints*.

[16] There's a very good example of having some idea of what a piece of text is about before you read it on pages 46 – 47 of McGuire (2015). Check it out.

- What definitions are there? How many of them are there? Will I have to remember them?
- $v = f\lambda$ is an equation. Is that the same equation as described in the section "The Equation of a Progressive Wave"? If not, what's the difference?
- What on Earth is *superposition*? Why is it important?
- What on Earth is *phase difference*? Why is it important? In fact, what is *phase* for that matter? What's the difference between *phase* and *phase difference*?
- Why are there different graphs denoting displacement against *distance* and displacement against *time*? What's the difference? Why are they important?

I could think of more, but that's plenty for now. All that you need to know at this stage is that doing this ask-yourself-questions-thing is really beneficial, before you start to read the stuff. You might not be very good at asking questions yet. You will improve, *fast*, once you start getting into the habit of doing this when you read.

And just as before, it's much better if you *write your questions down*.

So how can we get answers to these questions? Well...we start reading, of course!!

16.1.3 Recalling and Paraphrasing...

Once you have previewed the text, and you've generated some interesting questions that you would like answered, you are ready to read.

When you start, *only read one paragraph at a time*. Just read the first paragraph. Stop. Put the book down, and try to recall the ideas in the paragraph. Then put the information in that first paragraph *in your own words*. And it really helps if you *write this down*.

Now move on to the second paragraph, and do the same thing, except that when you paraphrase this paragraph, fold in the information from the first paragraph. Remember to recall what's in the paragraph before you try to paraphrase it.

Now move on to the third paragraph, and do the same thing, except that when you paraphrase this paragraph, fold in the information from the first two paragraphs...

...until you have paraphrased the whole piece.

So you think that's going to take a long time, right? McGuire states that *every single one* of the students that she has dicussed this method with reports that it was quicker than the method they had used previously. And that includes the method of just reading it!!

Using this method, there are no false starts. You move steadily through the text from beginning to end. Slow and steady wins the race. And another thing: once you've done this for a bit, you can get to fairly accurately estimate how long it will take you to do a piece of reading. That's also a useful thing to know.

16.1.4 ...Actively

The processes of recalling and paraphrasing, and the making of notes and outlines, should be done in a way that includes other successful strategies: the creation and use of memory aids, flashcards, mind maps, and concept maps; the generation of more questions; etc., etc.

And *actively* reading means that you also take advantage of your natural learning style. So whether you are a visual, aural/auditory, read/write or kinaesthetic/tactile learner, then you should just do what comes most naturally to you. I just want to make the point that when I spoke earlier about *writing stuff down*, what I actually meant was *record it somehow* that fits in with your natural learning style.

I'm primarily a visual learner, so when I make notes, yes, there is quite a bit of text, but I also draw lots of pictures: graphs, charts, tables, mind maps, concept maps, flow charts, etc., etc. I even do Integration by Parts[17] in a kind of picture. The idea is to actively record your learning session somehow. So if making a video or writing a play so that you can act it out later works for you, fine!!

Reading actively also means that if you are doing a quantitative course, and there are examples in the text, then *you should work through them*. This probably sounds really obvious now, and if it does, then perhaps you are grasping these ideas! I know of a series of Mathematics textbooks that essentially only have section titles, examples and questions in them. So *almost all* the teaching is done through examples. If you didn't do the examples, you wouldn't learn anything.

[17] Whatever that is!

16.2 Teaching the Material

Something else you might want to do in a *sprint* is to teach the material to someone.

We've already talked about this in Section 4.1. In trying to explain something to someone else, or even an inanimate object, you can often highlight gaps in your understanding.

It's a *really* good idea. Do it!

16.3 Solving Problems *Without* Using Examples as as Guide

Another thing that we often do when we are working on our own is to do homework. Or practice solving problems for a test or exam. And as the title of this section suggests, you should attempt to do your homework *without* using worked examples as a guide.

Counter-intuitive? Yes. Will this strategy give you enormous benefit? Most definitely: *YES*.

Normally when you do a homework on a new topic, you sit down with the questions, and you open the textbook to the appropriate section. You look at the first problem, and then look in the book for an example that's similar to the question, right? I know you do this because everyone does. I did.

This method is exactly the wrong way to go about doing homework problems.

Homework, *and example problems encountered in the textbook*, should always be treated as an opportunity for you to test yourself. Study for a homework the same way you would study for a test. Before looking at the questions, actively read the relevant section in the textbook, and/or your class notes. Work through the examples And do them *without referring to the given solutions*. As if you were in a test. Even if you get stuck, don't be tempted to sneak a peak at the solution before you've finished your "test".

Once you've worked through an example, check the answer. That's check the *answer*. Only. *Don't* check the *solution*. Only check the answer.

If your answer is wrong, then re-read the textbook section, and your notes on the topic, and go through the example again. But while you are doing the re-reading, see if you can figure out why you got stuck, or what mistake you made. *A great deal of deep learning happens during this investigation phase*.

Eventually (hopefully!) you arrive at the correct answer. Only then should you compare your *approach* to solving the problem with that of the book (or your teacher). If the approaches are different, are they both valid? Is your approach better? More understandable to you?[18] If so, maybe you should use your approach in future! If you're not sure whether your approach is valid, check with your teacher.

Quite often there's more than one way to solve a problem. And someone else's solution might not "click" with you. You might become locked into the way of thinking that what the book has done is the *only* way to solve the problem. That's bad! (See Section 6.2.)

Right. So we've read through all the material, and we've worked through all the examples, and we've got them all right. *Now* you can start to tackle the homework.

And of course: you tackle the homework in *exactly* the same way that you tackled the examples. Treat it as a test. Revise for the test until you think you are ready. Then take the test. Once you've looked at Question 1 (a), and you realise that you *aren't* ready for the test, then go and read the textbook section again...

It may occur to you that this way to tackle homework would take more time than you normally allocate to doing homework. That's true. It will. So don't leave homework to the night before (or the night after) it's due. Start tackling it straight away. Homework has been given to you to help you understand and assimilate the material. To get the most out of homework, follow this strategy.

[18] When I was at university, studying for my final exams, there was one course in particular that I couldn't get my head around. It was called "Materials", and I didn't understand any of it. Honestly. Neither did anyone else. The night before the exam I was looking over the stuff, and wondering how I could possibly answer any question on it. I noticed that on the whole module, there was only *one* mathematical derivation of an accepted formula. So I thought that there was a good chance that "Derive the formula for..." would come up as a question. But I had never understood the derivation before. What could I do? I had the idea that if I started with the initial assumptions, and used physics that *I understood*, together with *my own symbols for things*, then maybe I could get somewhere. So that's what I did. And about fifteen minutes later I'd come up with the formula!! *My way*!!

The next day, that question *did* come up. And of course, I did it my way! I couldn't answer any of the other questions, of course, and no-one else answered *any* question from that module.

I went to see the guy who taught the course a bit later, and he said "It's so refreshing to find someone that actually understands what I'm trying to teach them". I didn't have the guts to tell him the truth.

16.4 Creating Practice Exams

Another good idea for something to do in a *sprint* is to create practice exams. If you've got a test coming up, and you want to know what will be on the test, then you could always ask your teacher. If she tells you to "go away" (or something else which is semantically equivalent), then what could you do?

You could look in the textbook and the notes that you took in classes, and you could deduce what topics will come up on the test. And then you could go one step further, and *create your own practice test*! You can do this by looking at various questions from the textbook, or past exam papers. Groups are really good at doing this because you could assign each person in the group the responsibility for putting questions together for a particular topic. And that way *you* won't know what all the questions are going to be in advance!

Then, obviously, you could take your practice test! And take it in the same way as you do homework questions (See Section 16.3.)

The theory supporting this strategy is outlined in Section 10.

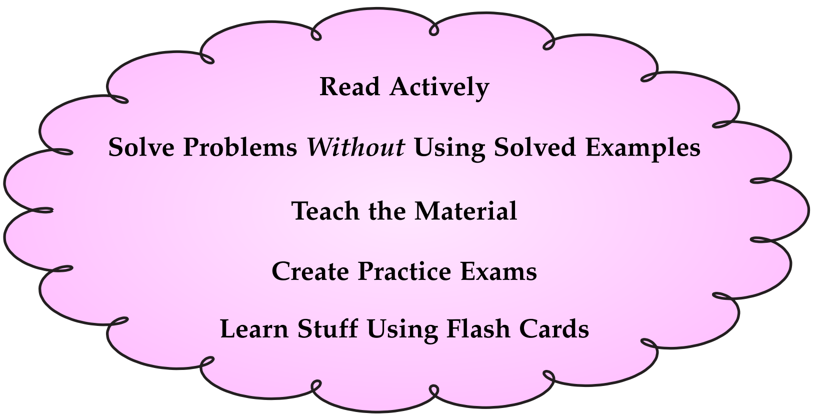
16.5 Learning Stuff

There will be occasions when you just have to learn some facts. In that case, try using a flash card system. They work. There are other techniques to improve memory, which I might talk about one day, but for now, let's keep it simple.

It's best if you make your own flash cards, as writing the cards helps the learning process. But there are very good electronic flash card systems that you can use on all your media devices. (See Section 8.)

16.6 Summary

Here, then, is a summary of the typical things that you will do during your *sprints*.



17 Mistakes

Are msitakes good or bad? You know what? It entirely depends on *when you make them*.

Mistakes are great if you don't make them on your final exam. Then they're pretty naff. But right up to that point, mistakes represent a golden opportunity (Zull (2012)). When you do your homework without access to the solutions (that are all too easily found on-line these days), and you make mistakes, you are going through the training program that your brain needs to be able to tackle the final exam without internet access.

So don't worry about making mistakes as you study. As long as they are *your* mistakes, and you have tried to tackle the problem meaningfully, then you will learn from the mistakes you make. But to do that, you need to analyse them. That means you need to explain (at least to yourself) what each mistake was, and what you should have done instead.

17.1 Analysing Mistakes

I would highly recommend keeping a little notebook in which you write an explanation of each mistake you make, when you make it. That's right: *all* of them.

Every time you have a test, or homework, or exam, have a look at the mistakes that you have made. Write them in your notebook.

Just going through the process of physically writing down what the mistake was, and what you should have done instead, is a really powerful motivator for not making so many mistakes in the future! Because it takes time and effort to write all this down.

But there are more important reasons for doing this:

- It cements in your mind the nature of the mistake. Writing down the mistake will reduce the chance of you making the mistake in the future, because just as you are about to make that same mistake again, your brain may well remember this moment now, and the nature of the mistake, and prevent you from making it again.
- It enables you to analyse all the mistakes you make *as a group*. That means you can see if there is a pattern to them. Do you continue to make the same kind of mistake time and time again? If so, try and do something concrete about trying to avoid the mistake. Take remedial action...

17.2 Taking Remedial Action

Here's a couple of examples of what I mean about taking remedial action. These are actions that I did when I was at school, to try and avoid making particular kinds of mistakes. Mistakes that I had identified I kept making by looking through my mistake notebook, and trying to spot patterns.

17.2.1 Doing One Thing at a Time in Algebra

When I started learning about how to solve equations in algebra, the idea that, in order to manipulate an equation, *you could do anything you like to the equation, so long as you do the same thing to both sides* came pretty quickly. To get from one line of working to the next I started thinking in terms of "What do I do to both sides next?". This is all very good, and I was very successful.

So successful in fact that I had another great idea[19]. Why not do *two* things to both sides at the same time?? That way I'll use less ink and paper, get to the answer quicker, and become a super-hero. Why not *three* things at the same time???

Of course, yes, I used less ink. Yes, I got to the answer quicker. Did I get the right answer? What do you think?

But I persisted with this ridiculous goal, of doing as much as I could with one line of working, for ages. It was only when I started keeping a record of the mistakes that I made that I realised that lots of times my mistake was that because I was doing so much in my head from one line of working to the next, I was often getting it wrong.

So, quite simply, I adopted the procedure of *only doing one thing at a time* when working through algebra.

When I adopted this strategy, I discovered several really important benefits:

- I started getting many more correct answers, which was of course the main benefit, but I also found that
- my work was easy to check, as I'd written everything down so it was easy to follow, and that
- when I was trying to explain what I had done to someone else (*teaching*, in other words), it was very easy *for them* to follow what I had done.

Win, win, win!

17.2.2 Limits on Integrals

If you know something about *integration* in Maths, and you have covered the integral transformation technique called "substitution", then check out Appendix B. There's something there you might find to your advantage.

[19] Or so I thought...

18 The Big Picture

So here's the big picture. It's an extremely abbreviated version of what you need to do to learn stuff.

The circles represent *sprints*: you could think of it a bit like a clock face representing an hour: the red bit shows the time when you are working ($\approx$ 40 minutes); the green bit shows the time when you're playing ($\approx$ 20 minutes)!

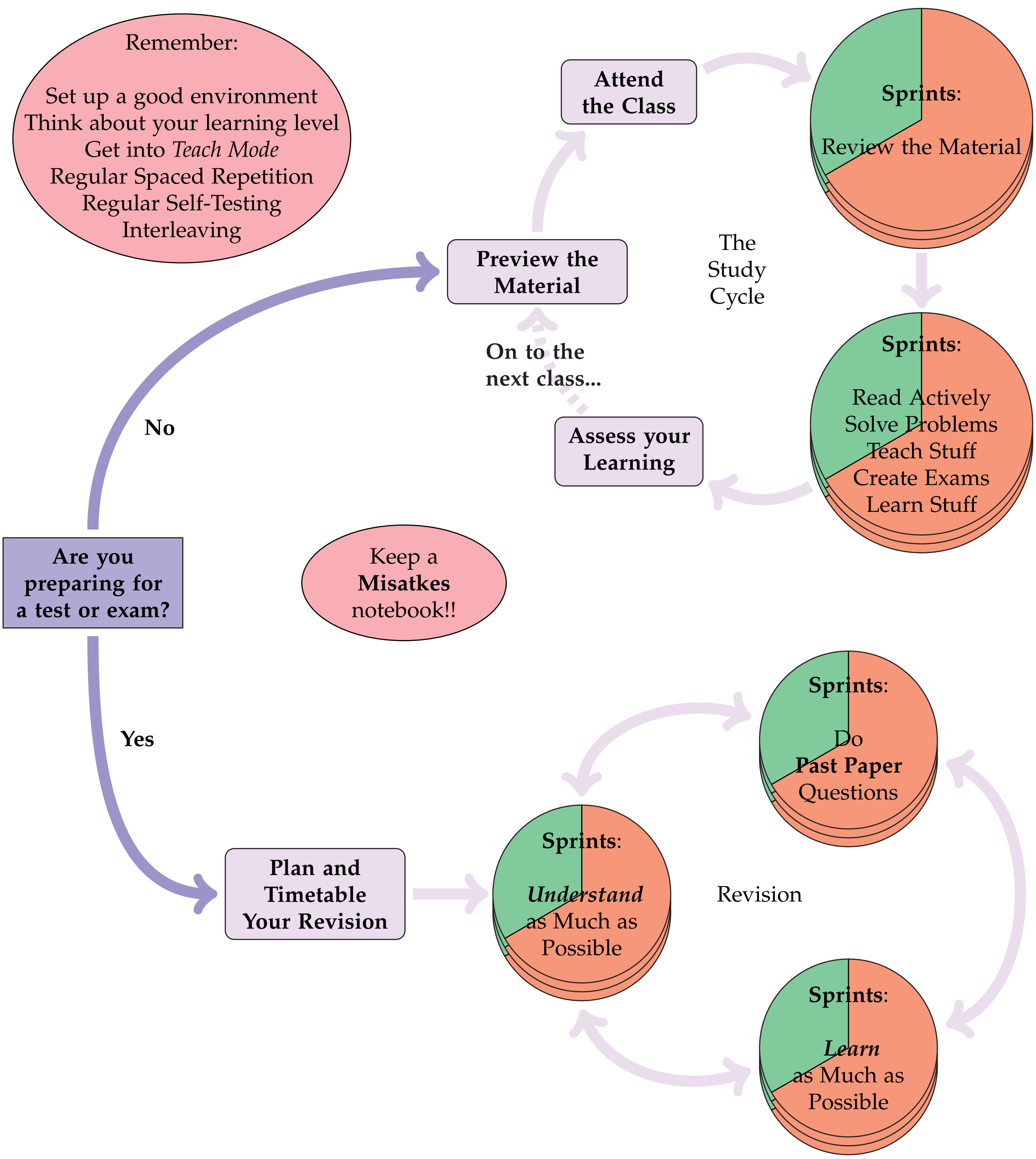


Figure 5: The Big Picture

A The Pythagorean Identity Revisited

This time around, I want to learn the following trigonometrical identities:[20]

$$\begin{aligned} \sin^2(x) + \cos^2(x) &\equiv 1 \\ 1 + \cot^2(x) &\equiv \text{cosec}^2(x) \\ \tan^2(x) + 1 &\equiv \sec^2(x) \end{aligned} \tag{2}$$

I could just learn the equations and hope I remember them when it comes to exam time. But as I've said, for me that's not the greatest plan[21].

Instead, how about learning this *picture*: I don't know about you, but I find learning pictures easier than

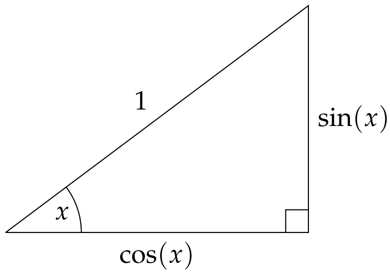


Figure 6: My Replacement for SOH CAH TOA! (Revisited)

learning words and equations. Now if we know this picture, then *we can work out what all the three equations are that we need to learn*. Let's see how we can do this. First, we know Pythagoras' Theorem. And our picture is of a right-angled triangle, so we can just use that directly. That gives us the first equation:

$$\sin^2(x) + \cos^2(x) \equiv 1 \tag{3}$$

Now to get the other equations, instead of learning them, we can use what we already know. Let's start with Equation (3), and divide both sides of the equation by $\sin^2(x)$:

$$\begin{aligned} \frac{\sin^2(x)}{\sin^2(x)} + \frac{\cos^2(x)}{\sin^2(x)} &\equiv \frac{1}{\sin^2(x)} \\ \Rightarrow \quad 1 \quad + \cot^2(x) &\equiv \text{cosec}^2(x) \end{aligned}$$

And if start with Equation (3), and divide both sides of the equation by $\cos^2(x)$:

$$\begin{aligned} \frac{\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} &\equiv \frac{1}{\cos^2(x)} \\ \Rightarrow \quad \tan^2(x) + \quad 1 \quad &\equiv \sec^2(x) \end{aligned}$$

So we have a choice: we can either try and learn the equations (2) directly, as three unrelated facts, or we can learn the picture (Figure 6) and a *process*. The three equations (2) are known as the *Pythagorean Identities*. It's not hard to see why. They are all equivalent to Pythagoras' Theorem through Figure 6 and a bit of algebra. Learning the picture and the process shows a lot more mathematical sophistication, I think. It shows a level of *understanding*, doesn't it?

A.1 Understanding...

I have been teaching mathematics now for over 40 years, and I have never learned equations (2). I don't need to[22]. I know where they come from. So if I need one of them any time, I just work it out.

Another thing that's really interesting about equations (2) is that they divide the six fundamental trigonometrical functions, $\sin(x)$, $\cos(x)$, $\sec(x)$, $\tan(x)$, $\text{cosec}(x)$, and $\cot(x)$ into *three pairs* of functions: (1) $\sin(x)$

[20] If you've never come across $\cot(x)$, $\text{cosec}(x)$ or $\sec(x)$ before, they are defined like this: $\cot(x) \equiv \frac{1}{\tan(x)} \equiv \frac{\cos(x)}{\sin(x)}$, $\text{cosec}(x) \equiv \frac{1}{\sin(x)}$ and $\sec(x) \equiv \frac{1}{\cos(x)}$. Why we need these things, I'll never know. We could so easily do without them altogether.

[21] There are *worse* plans. The Scottish army deciding to invade England during the Black Death, for example.

[22] So I can keep my brain clear so that it can remember useful information, like who won the F.A. Cup in 1948.

and $\cos(x)$, (2) $\sec(x)$ and $\tan(x)$, and (3) $\operatorname{cosec}(x)$ and $\cot(x)$. It turns out that these pairs of functions are related in other ways too. By differentiation and integration, for example. So we are forging links between different areas of our subject. We are forming patterns. *Chunks*, as Barbara Oakley calls them. General principles that we can use in other situations.

And while we're on the subject, there's another thing I'd like to get off my chest: I called Figure 6 "My Replacement for SOH CAH TOA!" for a good reason. I never use SOH CAH TOA. It's crap. All you need is Figure 6[23]. It's interesting that Figure 6 is actually the *definition* of $\sin(x)$ and the *definition* of $\cos(x)$. SOH CAH TOA is built on top of Figure 6. But Figure 6 is easier to use than SOH CAH TOA!! So why do we teach something artificial, something that's only been created to help learn Figure 6, when learning the picture is easier???

B Limits on Integrals

When you first start to integrate functions, you come across the idea of *limits* on integrals. These are normally written at the top and the bottom of the integral sign like this:

$$\int_0^2$$

and what that means in something like this

$$\int_0^2 (x+5)^2\, dx \tag{4}$$

is that 0 and 2 are values *of* x. Every book you have ever seen, and every exam, write limits this way.

The problem with this is that when you start learning about integral transformation techniques such as "substitution", where the idea is that you transform the integral by changing the variable from x to something else, then you open up the way for a specific type of mistake.

When you transform an integral like (4), you do it by introducing a new variable, u, say, that you hope will transform the integral from something you can't integrate, into something you can integrate. And you could do that like this. Let $u = x + 5$. Then when you go through the motions, and you transform the function (the $(x + 5)^2$ bit), and you transform the differential (the dx bit), into u-stuff you get

$$\int_0^2 u^2\, du \tag{5}$$

And at this point it's really easy to fall into the trap of thinking that you have done all the transforming you need to do. As I very often used to do. But those limits are *values of x, not u!* We have to transform the limits as well! And when you write your limits as in (5) (as everyone does), then you can easily fall into this trap.

However, if you wrote your limits like this

$$\int_{x=0}^{x=2} (x+5)^2\, dx$$

(showing very clearly that 0 and 2 are both values of x) then after transforming the function and the differential, you get

$$\int_{x=0}^{x=2} u^2\, du$$

and it's really clear that you have not transformed your limits yet. You have vastly reduced the possibility of making the I-forgot-to-transform-my-limits-again mistake.

[23] And the definition of $\tan(x)$, actually.

Notes

[1]This is an endnote.

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