Model Sensitivites

Contents

Prerequisites

You should read my paper called "How to Intonate a Guitar" (Smith (2014)) before reading this piece.

Notes

None.

Document History

Date	Version	Comments
26th July 2014	1.0	Initial creation of the document.

1 Introduction

When using the model described in (Smith, 2014), it is quite difficult to acquire perfect knowledge of all the required data, particularly the string data.

It would be useful, then, to have some idea of the consequences of estimating some of the data in the model. This paper outlines the effect on the nut and bridge positions of tuned strings of altering each of the variables used in the model over a range of values, while keeping all the other variables constant.

The base set of data is shown in Table 1.

Table 1: Quintero String Data

	First String	Second String	Third String
Open Note	G	D	G
Open Frequency (Hz)	196.000	146.832	97.999
Core Diameter ($\times 10^{-6}$ m^2)	3500	3500	3500
Young's Modulus (GPa)	200	200	200
Density ($\times 10^{-5}$ kg m^{-1})	115	258	469
Ln (mm)	140	110	75
Lb (mm)	45	45	45
Fret Height (mm)	1.5	1.5	1.5
Nut Height (mm)	2.0	2.0	2.0
Bridge Height (mm)	4.0	4.0	4.0
f_H	0.3	0.3	0.3
f_R	0.8	0.8	0.8
Scale Length (mm)	640	640	640

The data in table 1 was taken from a guitar that I made myself, which I call my "Quintero" guitar. Hence the unusual *Scale Length*. The guitar was tuned G-D-G using the 3rd, 4th and 5th strings from a standard packet of six electric guitar strings. The *Open Frequency*, the frequencies of the notes required at the open strings, L_n, L_b, *Fret Height*, *Nut Height*, and *Bridge Height* were measured values; *Core Diameter*, *Density*, f_H and f_R are estimated values.

2 Results

2.1 Varying f_H

f_H and f_R are parameters that are used to model the guitarist. The way a guitar is played could vary significantly, so these are important factors with which to perform a sensitivity analysis.

Keeping all other values constant, f_H was varied using values in the range 0.1 to 0.5. In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figure 1a.

It comes as no surprise to discover that the greater the force a player uses to fret a note (the greater the value of f_H), the more the string will be pulled out of tune, and the greater the nut and bridge offsets need to be.

2.2 Varying f_R

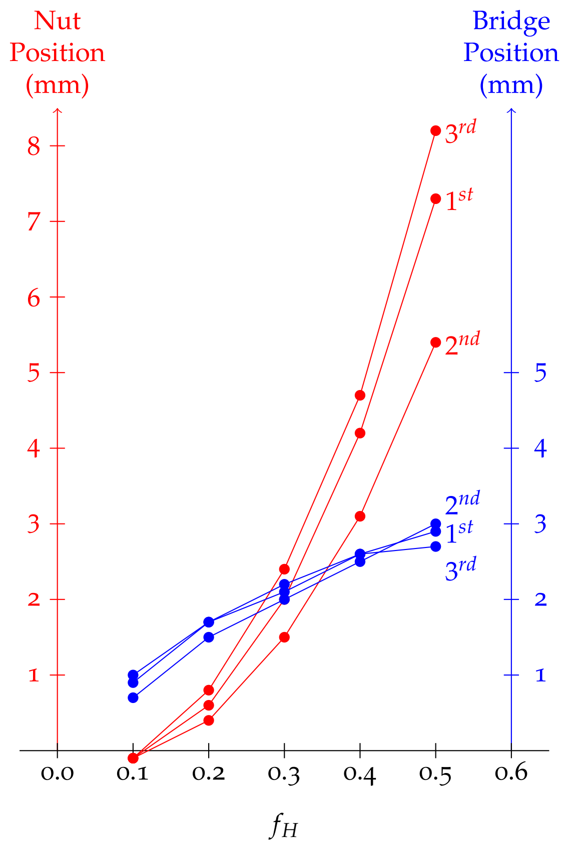
Keeping all other values constant, f_R was varied using values in the range 0.4 to 0.9. In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figure 1b.

Figure 1b shows that it is better to fret a note around 0.7 of the way from the previous fret to the required fret.

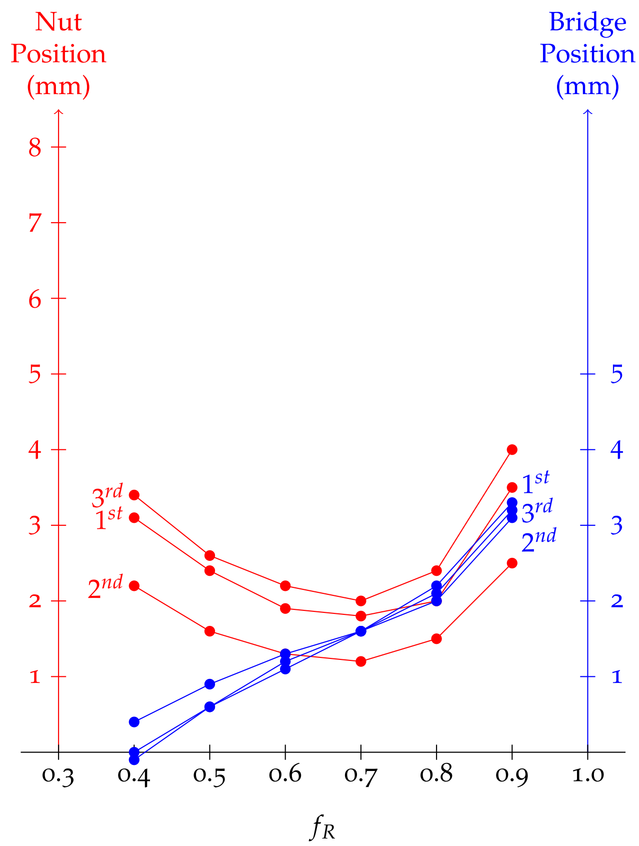
The way that f_R is modelled could be altered so that it was not constant for every fret. Since the spacing between frets is larger nearer the nut, perhaps f_R being a function of fret number may lead to interesting observations as to the way a guitar is played.

Figure 1: Model Sensitivites: f_H and f_R

(a) Varying f_H



(b) Varying f_R

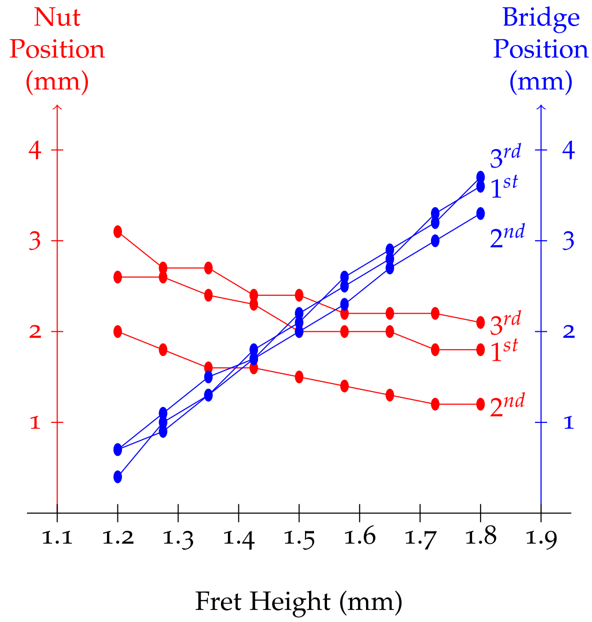


2.3 Varying Fret Height

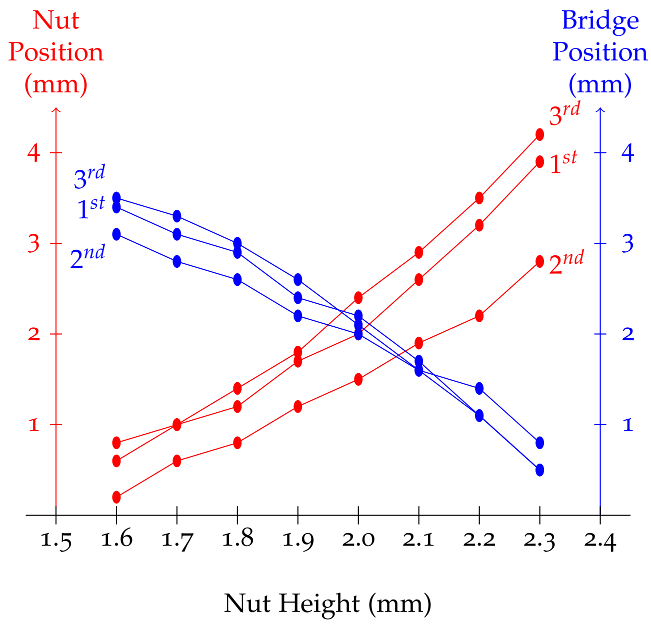
Keeping all other values constant, the height of the frets was varied using values in the range 1.2 to 1.8 mm. In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figure 2a.

Figure 2: Model Sensitivites: Fret Height and Nut Height

(a) Varying Fret Height



(b) Varying Nut Height



Interestingly, nut offset is relatively unaffected by fret height, whereas there is a larger effect on bridge offset.

2.4 Varying Nut Height

Keeping all other values constant, the nut height was varied using values in the range 1.6 to 2.3 mm. In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figure 2b.

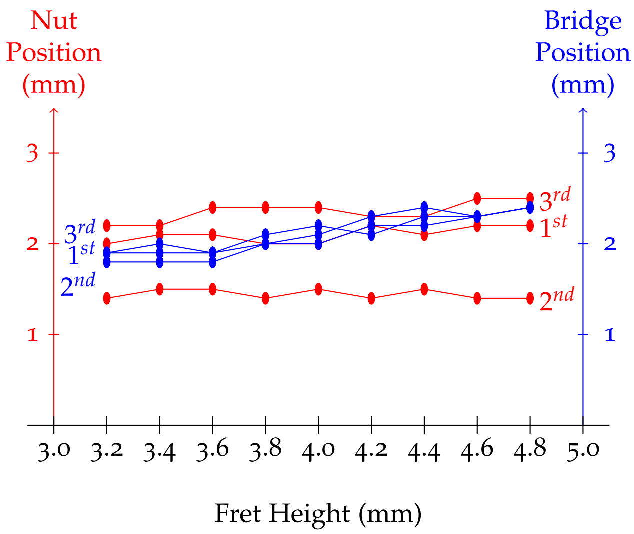
Nut height has a marked effect on nut and bridge offsets, and consequently how well a guitar string is in tune. The smaller the nut height the better!

2.5 Varying Bridge Height

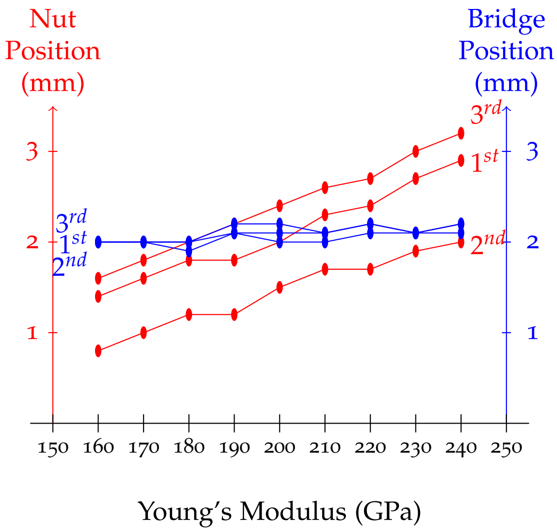
Keeping all other values constant, the bridge height was varied using values in the range 3.2 to 4.8 mm. In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figure 3a.

Figure 3: Model Sensitivites: Bridge Height and Young's Modulus

(a) Varying Bridge Height



(b) Varying Young's Modulus



Both nut and bridge offsets seem to be relatively unaffected by varying the height of the bridge.

2.6 Varying Young's Modulus

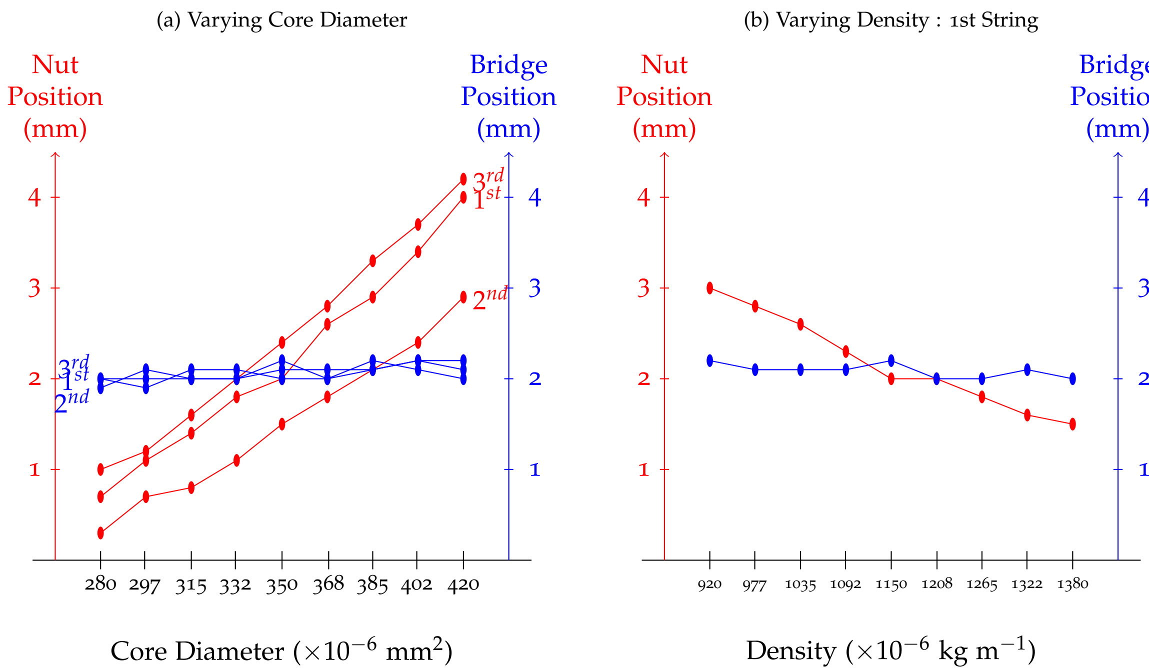
Keeping all other values constant, the Young's Modulus of the string core material (assumed to be steel) was varied using values in the range 160 to 240 GPa. In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figure 3b.

Estimating the Young's Modulus for the core material of a string accurately is important, as Young's Modulus affects the nut offset. The bridge offset does not seem to be affected by Young's Modulus to any significant extent.

2.7 Varying Core Diameter

Keeping all other values constant, the diameter of the core material of the string was varied using values in the range 280 to 420 $\times 10^{-6}$ mm^2 for the cross-sectional area of the string. In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figure 4a.

Figure 4: Model Sensitivites: Core Diameter and Density

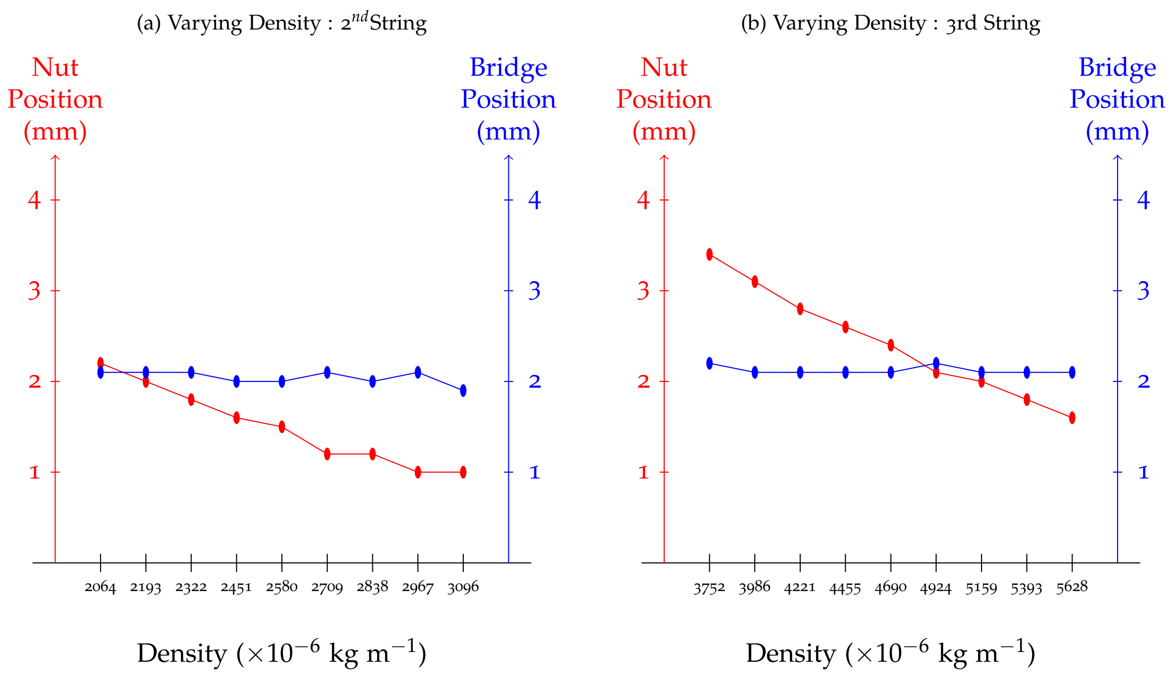


Estimating the diameter of the core material of a string accurately is important, as it affects the nut offset. The bridge offset does not seem to be affected by core diameter to any significant extent.

2.8 Varying Density

Keeping all other values constant, the density of the string was varied using values in the range 920 to 5628 $\times 10^{-6}$ kg m^{-1} (dependent on the string). In each case, the string was tuned using the model so that each string was tuned to within 3.5 cents of the correct frequency at each fret, and the nut and bridge positions were obtained. The results are shown in Figures 4b, 5a and 5b.

Figure 5: Model Sensitivites: Density



Estimating the density of a string accurately is important, as it affects the nut offset. The bridge offset does not seem to be affected by string density to any significant extent.

References

Smith, S. (2014). How to intonate a guitar.