Guitar Fret Theory

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Contents

Prerequisites

None.

Notes

None.

Document History

Date	Version	Comments
8th June 2014	1.0	Initial creation of the document.
17th September 2014	1.1	Removing the mathematical model, adding the sections on intervals, general tidying up, etc.

1 Introduction

The reason I wrote this little document was because I make guitars, and I wanted to investigate where I should put frets when making a fretted fingerboard. I wanted to learn about the theory of fret positioning, and I wanted to find out about any practical concerns that might ensue.

For example, one of the first guitars I made had fret positions determined by a fret template. So their positions were pretty accurate. However, fretting strings near the nut produced notes that were a bit sharp. And I couldn't get the tuning at those frets exactly right (while keeping all the other frets in tune!), no matter how low I cut the nut slots. So what's going on?

2 Notes, Frequencies and Intervals

2.1 Notes and Frequencies

Before we can talk about where fret positions need to be on a guitar, we have to know a bit about waves. About how waves occur on plucked strings, and how the frequency of those waves depend on the length of the available string on which they are travelling. This is standard A-level physics stuff, so I won't go into all the detail here. If you want to find out more, do an internet search, or look up any decent A-level physics text book, such as Nelkon and Parker (1988).

Check out, for example, Figure 1.

Figure 1: The Fundamental Frequency (First Harmonic)

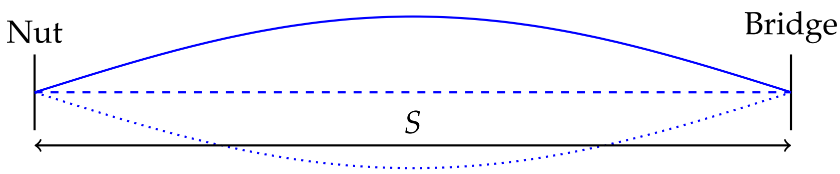


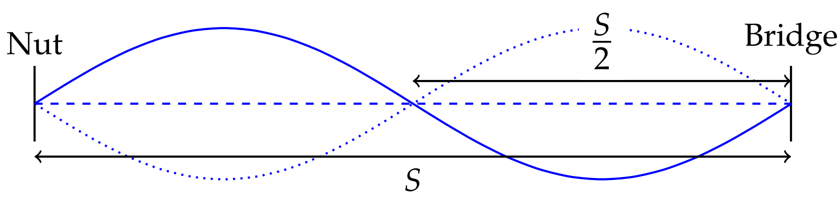
Figure 1 shows a schematic of the string vibrating in what is known as the fundamental mode. The blue dashed line represents the unplucked string; the solid and dotted lines represent two snapshots of the string as it is vibrating after being plucked. The two ends of the string are fixed, so they don't vibrate at all; the rest of the string is vibrating with the largest wavelength possible (a length of $2S$). This mode of vibration has the largest wavelength possible for the string this length. So this must correspond to the lowest frequency possible for a string this length.

But vibrating strings are complicated things, and at the same time that a string is vibrating in fundamental mode, it can also be vibrating as shown in Figure 2: this is known as the *Second Harmonic* for this string. The wavelength of the vibrations this time are S, and the frequency of the vibration is double that in Figure 1.

And the string can vibrate in lots more modes than this. *Simultaneously*. In fact, it's the combination of the ways a string vibrates when you pluck it that makes an instrument sound the way it does. But that's another story...

The interesting thing to get out of all this from a fret-positioning point of view, is that when you double the frequency of a note, you go up an octave. For example, The A below $middle - C$ corresponds to a frequency of vibration of $220Hz$. This note is known in the trade as $A3$. If you

Figure 2: The Second Harmonic



double the frequency, to $440Hz$, the note you hear is another A: it's the A that is one octave above $A3$. And to distinguish it from $A3$, it is known as $A4$.

If you doubled the frequency again, to $880Hz$, then you'd get the A one octave up from $A4$: that is known as $A5$, etc. And if you *halved* the frequency of $A3$, to $110Hz$, you'd get $A2$. $A1$ has a frequency of $55Hz$. Get the idea? By the way, the reason that the $27.5Hz$ A is $A0$, the *start* of the sequence, is that human hearing isn't very sensitive to frequencies much lower than this.

This doubling-the-frequency-to-go-one-octave-up thing works the same for all the notes in the scale, not just the As.

2.2 The Western Scale

Now the way that the Western scale is constructed, there are 12 notes in an octave, and they are *equally spaced* within the octave. And therein lies a tale.

It turns out that *equally spaced* doesn't mean, for exmaple, $110Hz$, $140Hz$, $170Hz$, $200Hz$, etc, where to get the next note you *add* so many Hertz. No. It means that instead of *adding* the same something each time, we need to *multiply* by the same something each time.

For example, $A2$ has a frequency of $110Hz$. It turns out that the next note up, $A\#2$ has a frequency of $116.54Hz$. Now to get 116.54 from 110 by multiplying by something, you have to multiply by $\frac{116.54}{110} = 1.0594....$

And the next note up from $A\#2$, $B2$ has a frequency of $123.47Hz$. Now to get 123.47 from 116.54 by multiplying by something, you have to multiply by $\frac{123.47}{116.54} = 1.0594....$ And so on.

Now where does this magic 1.0594... number come from? Well, if you think about it *very* hard, if you need to go up 12 notes in an octave, and remember that the same note in the next octave has double the frequency, then to get to that next octave note from the current one, you would have to multiply the frequency of the current note by the same something, *12 times*.

Let's let that something be r. Then, for example, to get the frequency of $A3$ (which we now know to be $220Hz$), knowing the frequency of $A2$ is $110Hz$, you'd have to do this:

$$110 \times r \times r \times r \times r \times r \times r \times r \times r \times r \times r \times r \times r = 220$$

And if you know a bit of maths, we can simplify this to

$$110 \times r^{12} = 220$$

Dividing both sides by 110:

$$r^{12} = 2$$

And taking the 12-th root of both sides (of course!),

$$r = 2^{\frac{1}{12}} = 1.059463094...$$

2.3 Intervals

Here, and in Section (3) I am basically following Harkleroad (2006).

An interval is just the musical "distance" between two notes. You could measure in interval in various ways: for example by using the difference in frequency between two notes. This turns out to be quite a bad idea, because it would be nice if the interval say between $A2$ and $B2$ was the same as the interval between $A3$ and $B3$. If you used frequencies, you would get different values for the interval (try that out now! You should have enough information in Section 2.1 to be able to do this.).

Instead, in the West, where we use the 12-note equal-temperament scale, the interval between two notes is normally determined by the number of frets (I can't use the word "note" here!) between the two in question. As an example, the two sounds that constitute the two "Twinkle"s in "Twinkle, Twinkle Little Star" are seven frets apart. The second "Twinkle" is seven frets higher than the first (whichever fret and string you start with).

So - what do you think that this interval is called? A seven? A seventh? No. It's called a *fifth*!!! The reason is that the second sound is five "notes" of a major scale up from the starting note (including both starting and end notes!!!). In musical terminology, there are *eight* notes in an octave, not *12*. Between some of the notes, are *half-notes*. Quite bizarre. From here on in, I'm going to use the term "note" to refer to the sound that can be played at any fret on any string. So in the rest of this document, there are seven notes in a fifth. Confused, or what?

I don't really want to get into all this naming stuff here. I'm much more interested in the ideas. And here's the big one: some pairs of notes sound nice when you play them together, and one after another. The pleasantest pair of notes are two an octave (12 notes) apart. The next nicest are two notes a fifth (7 notes) apart.

Now the two notes an octave apart have the special 2-1 frequency relationship. We've already seen that. What about two notes a fifth apart? In fact, you can work out for yourself that two notes a fifth apart have a ratio of frequencies of $\frac{3}{2}$. A nice simple ratio.

Another important (nice-sounding) interval is called a $fourth$, which is an interval of 5 notes. A fourth complements a fifth because if you go up a fifth, and then go up a fourth, you've gone up an octave. Now if you work out the frequency ratio with two notes a fourth apart, it turns out that their frequency ratio is $\frac{4}{3}$. Another simple ratio.

Other important intervals in Western music are: the *major third* (spanning four notes), with a frequency ratio of $\frac{5}{4}$; the *minor third* (spanning three notes), with a frequency ratio of $\frac{6}{5}$; the *major sixth* (spanning 9 notes), with a frequency ratio of $\frac{5}{3}$; the *minor sixth* (spanning 8 notes), with a frequency ratio of $\frac{8}{5}$. I could go on.

Forgetting about the odd naming conventions involved, and just sticking to the maths, then this isn't quite so bad. The upshot is that nice sounding combinations of notes have simple ratios of frequencies. And that will be all be down to the way harmonics work. OK, so what's the problem...?

3 The Problem With Intervals

The problem is that, for example, you cannot make all octaves $\frac{2}{1}$, and all fifths $\frac{3}{2}$ *at the same time*.

Here's an example of an incompatibility. Go up an interval of 14 notes. You could do that by going up a fourth and then a (major) sixth; you could go up by two consecutive fifths, etc. etc. (See the table below.)

Interval	**Frequency Ratio**	**Separation**
(Minor) Third	$\frac{6}{5}$	3
(Major) Third	$\frac{5}{4}$	4
Fourth	$\frac{4}{3}$	5
Fifth	$\frac{3}{2}$	7
(Minor) Sixth	$\frac{8}{5}$	8
(Major) Sixth	$\frac{5}{3}$	9
Octave	$\frac{2}{1}$	12

Let's see what happens to frequency ratios if we travel these two routes.

First, let's go up 14 notes by going up a fourth and a (major) sixth. OK, let's say that the note we started from had a frequency f. Then if we go up a fourth, then that note will have a frequency f_1 of

$$f_1 = \frac{4}{3} \times f$$

Now we go up a (major) sixth from that note. If the frequency of the the note we get to is f_2, then it will have a frequency of

$$f_2 = \frac{5}{3} \times f_1 = \frac{5}{3} \times \frac{4}{3} \times f = \frac{20}{9} f$$

Now let's go up the same 14 notes by going up two consecutive fifths. We're starting from the same note, with frequency f. So if we go up a fifth, then that note will have a frequency f_1 of

$$f_1 = \frac{3}{2} \times f$$

And now we go up a fifth from that note. If the frequency of the the note we get to is f_2, then it will have a frequency of

$$f_2 = \frac{3}{2} \times f_1 = \frac{3}{3} 2 \times \frac{3}{2} \times f = \frac{9}{4} f$$

Now as decimals, $\frac{20}{9} = 2.222...$ and $\frac{9}{4} = 2.250$. Now these are close, but *they are not the same*!! And this is very important, because two notes with similar frequencies, but different, tend to sound very bad.

So something's got to give. A compromise is required. To end up with *exactly* the same frequency jump when you go up a particular interval in different ways, you will have to change the frequency ratios of some of the basic intervals. For example, if we want to keep the fifth frequency ratio (because it sounds so good), and we want to keep the fourth ratio (because that sounds so good too), then we would have to change the ratio of the (major) sixth so that we ended up with a combined ratio of $\frac{9}{4}$ by traversing the interval both ways.

See Harkleroad (2006) for much more on all this.

Now what on Earth does this mean for the positioning of frets on a guitar fretboard? It means that, even though in Section (4) we develop what we think are perfect fret placements, in effect, there will be small frequency errors on most frets, because the fret positions do not take any of this compromising-on-intervals-business on board. And it would be pretty difficult to, if the idea of a guitar is an instrument that can play any song in any key.

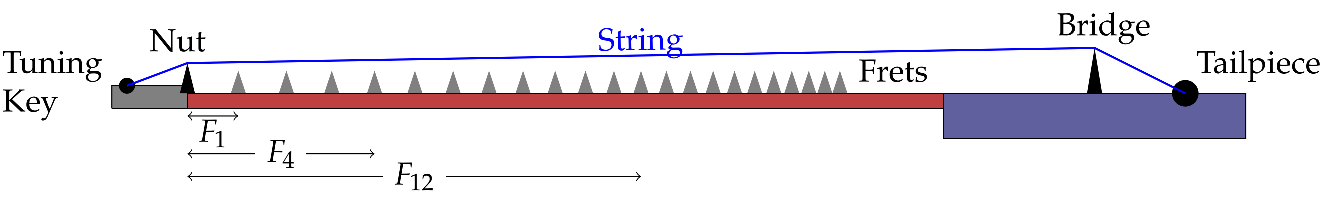
4 The Positioning of Frets

First, let's define a few symbols that I'm going to use in the following notes.

Let's say that we have a guitar with a scale length of S. That's the distance between the nut and the bridge.

Let's call F_k the position of the k^{th} fret, starting numbering the frets at the nut. So the first fret will be at a distance F_1 from the nut, the second a distance of F_2 from the nut, etc. Using this terminology, you could think of the nut being at fret position F_0. See Figure 3.

Figure 3: The Guitar



So, from Section 2 we know that:

- to double the fundamental frequency of a string you have to halve its length;
- a note an octave up from a given one has twice the frequency;
- the Western scale is divided into 12 equally spaced notes.

4.1 The 12^{th} Fret

Putting these three facts together we can see that holding a string down in the middle of its length will double its frequency, and hence give you a note an octave up from the full length string. That also means that the position in the center of the string must correspond to the 12^{th} fret: the center of the string must be 12 notes up from the base note, to give you a full octave up.

So it's easy to position the 12th fret: it's always half way between the nut and the bridge. But what about the other frets?

4.2 The Other Frets

Well, remember that to get the next note up we have to multiply the frequency of the note by this magic number, r? Well then, we have to position the frets so that this happens. And here's how we do that.

There is a formula that relates the velocity v of a wave on a string that has wavelength λ and frequency f. It is this:

$$v = f\lambda \tag{1}$$

This is the equation that relates a characteristic length to the frequency of a wave. And this formula is useful because we need to be able to convert frequencies into lengths, in order to position frets properly.

Another fact that we need in order to do this analysis is that the speed of a wave up and down a string that you get when you pluck the string does not depend on the frequency or the wavelength of the wave[1].

OK, right. So here we go.

Looking at the wave that's set up when you pluck an open (unfretted) string (see Figure 1) then, using Equation (1) the fundamental frequency f_0 will be related to the scale length S of the string by:

$$v = f_0 \times 2S$$

So

$$\frac{v}{2} = f_0 S$$

Now since v is constant, then $\frac{v}{2}$ is constant. And if the left-hand side of this equation is constant, then the right-hand side will be constant. And here is how we relate fret positions to the frquencies of the notes: if f is the frequency of the note we want to play, and L is the length of the string we need to play this note, then

$$fL = \text{constant} \tag{2}$$

Now if we are fretting from the *nut* (as is usually the case) then the L will represent the distance from the bridge to the fret. Let's think about the first fret position, then. Remember that for the first note up from the fundamental, we need the frequency (f_1) to be:

$$f_1 = rf_0$$

Well that means that we will have to reduce the length of the string (to L_1) to get this note, and the new length will have to be $\frac{S}{r}$ because

$$f_1 \times L_1 = rf_0 \times \frac{S}{r} = f_0 S = \text{constant}$$

Now for the second note up from the fundamental, with a frequency of f_2, then

$$f_2 = rf_1 = r^2 f_0$$

and to get a note with frequency $r^2 f_0$ we will need a length L_2 of $\frac{S}{r^2}$ because

$$f_2 \times L_2 = r^2 f_0 \times \frac{S}{r^2} = f_0 S = \text{constant}$$

And in general, then, the distance of the k^{th} fret from the bridge will be

$$L_k = \frac{S}{r^k}$$

But we normally measure fret positions from the *nut*, rather than the *bridge*. But of course, if L_k is the bridge-fret distance of fret k, then the nut-fret distance F_k will just be $S - L_k$. So:

$$F_k = S - L_k = S - \frac{S}{r^k}$$

or

$$F_k = S\left(1 - \frac{1}{r^k}\right) \tag{3}$$

Now this is the magic fret positioning formula that (almost!) all guitars ever made have used to position the frets on the fingerboard. It's so important, I think I'll emphasize it a bit:

There. that's nice.

[1]To be honest, that's not absolutely true. It does vary a bit, primarily depending upon the physical characteristics of the string, but for our purposes, we can assume velocity is constant.

$$F_k = S\left(1 - \frac{1}{r^k}\right)$$

4.3 Consequences

Check this out. If we look at the distance between the first and second frets, $F_2 - F_1$, then

$$\begin{aligned}
F_2 - F_1 &= S\left(1 - \frac{1}{r^2}\right) - S\left(1 - \frac{1}{r}\right) \\
&= S\left(\frac{r^2-1}{r^2} - \left(\frac{r-1}{r}\right)\right) \\
&= S\left(\frac{r^2-1}{r^2} + \frac{r-r^2}{r^2}\right) \\
&= S\left(\frac{r-1}{r^2}\right) \\
&= \frac{1}{r}S\left(1 - \frac{1}{r}\right) \\
F_2 - F_1 &= \frac{1}{r}F_1
\end{aligned}$$

So the distance from the first to the second fret is $\frac{1}{r}$ times the distance from the nut to the first fret. And it turns out that this is true in general: that is the spacing of the frets goes down by the factor $\frac{1}{r}$ each time.

A major consequence of this is that if you have a fret template, with a scale length of S, then you can use the same template to give you *different scale lengths*. If you place your nut at the first fret *of the template*, then the distance to the first fret *on the guitar*, F_1' will now be

$$F_1' = \frac{1}{r}F_1 = \frac{1}{r}S\left(1 - \frac{1}{r}\right)$$

using Equation (3), so

$$F_1' = \left(\frac{S}{r}\right)\left(1 - \frac{1}{r}\right)$$

which is the normal first fret distance *for a scale length of* $\frac{S}{r}$!!

So, for example, if you had a 647.7 mm scale length template (i.e. a template for a Fender Stratocaster), then by putting the nut at the first fret position, and then marking off your frets starting with the first fret at the template's second fret, you would be constructing a guitar with a scale length of $\frac{647.7}{r} = 611.3$ mm.

This is handy (ha, ha!) because...

5 Hand Size and Scale Length

If you want a child to play a guitar, there's no point giving her a full sized guitar because her hands are too small to play it. That seems obvious. Children's guitars are quite often labelled as $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ size. But what does that mean? Obviously they are shorter than an adult guitar, but there is no standardisation of scale length. Typically, scale lengths start from around 18 inches (about 460 mm) for a $\frac{1}{4}$ size guitar, and rise continuously to the approximately 25 inches (around 640 mm) for an adult guitar.

So if it's obvious that a child with small hands will find it difficult to play a full-sized guitar[2], then wouldn't it be obvious that an adult with small hands would find it difficult to play a full-sized guitar? Shouldn't a player choose a guitar sized appropriately for them? There is a lot of online discussion about this topic. See, for example, kechance et al. (2012).

It is very convenient for manufacturers to only make a particular model of guitar in only one size. And that's what you find throughout the industry. You can't get Fender Stratocasters with scale lengths other than 25.5 inches. And you can't get a Gibson SG with a scale length other than 24.75 inches. It's just not possible[3]. Unless you've had the guitar especially adapted.

But if you are building your own guitars...

5.1 The Thumb - Little Finger Span

A (relatively) simple model that has been used to assess the appropriate scale length for a hand of a given size is to use the span between the tip of the little finger to the tip of the thumb when the hand is stretched out (kechance et al. (2012)) to determine the "size" of the player's hand. From now on, I will refer to this distance simply as "the span". This is not universally accepted as a model of hand size, but it gives us a start. Something to play with. A rule of thumb, you might say (oh dear).

A luthier performed his own experiments to decide what scale lengths were appropriate for his students using thumb tip - little finger tip distance as a measure of hand size, and came up with results shown in Table 1.

Table 1: Hand Size and Scale Length

Thumb to Little Finger Span (mm)	Appropriate Scale Length (mm)
$<$170	615
170 to 190	630
190 to 210	640
210 to 230	650
230 to 250	656
$>$250	664

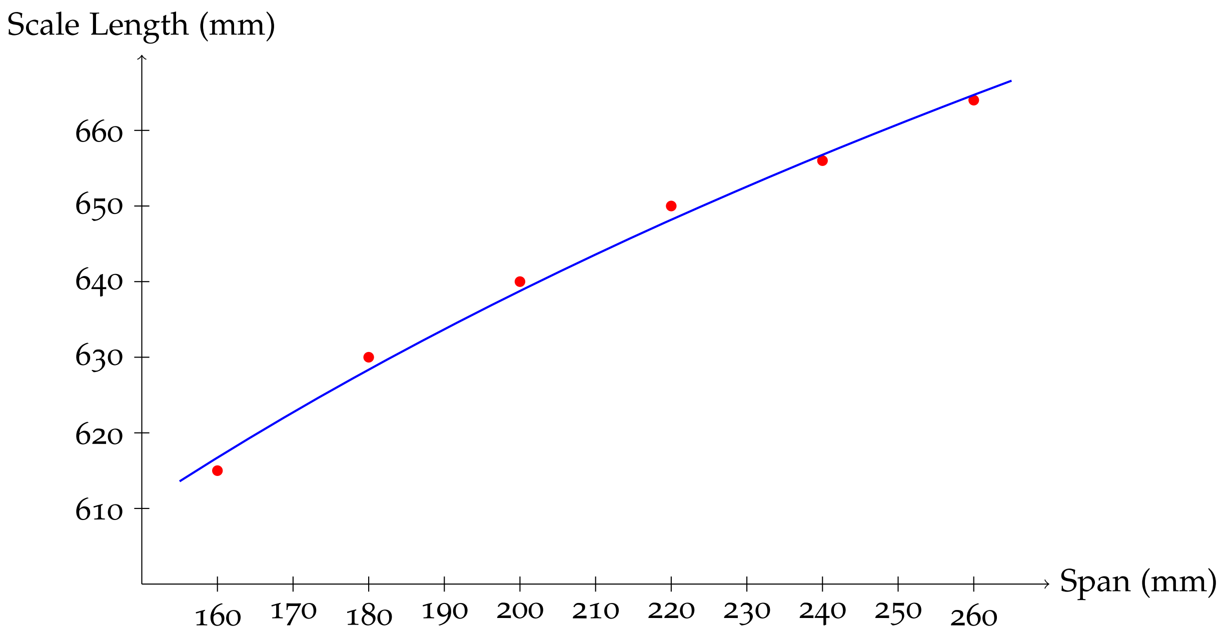
Now I appreciate this is all very seat-of-the-pants stuff, but let's run with this. From the data shown in Table 1, I have assumed a value of 160 mm for the $<$170 span data point, a value of 260

[2] Whatever a "full-size" guitar is!

[3] There are exceptions to every rule, but you get my point, I hope.

mm for the >250 span data point (all very arbitrary, I know), and have drawn the graph shown in Figure 4. The data are shown as red dots in Figure 4. I then drew a line of best fit through this data, using a logarithmic regession model. That's the blue line in Figure 4. I used a logarithmic model for no reason other than the pattern of the dots reminded me of a log graph.

Figure 4: Hand Size and Scale Length Graph



The formula for the line of best fit turned out to be

$$\text{scale length} = 98.8 \times \ln(\text{span}) + 115.3 \text{ mm}$$

Now the detail of the mathematics here is completey irrelevant. What is important is that we now have a method for estimating the best scale length for a given person.

My span is 205 mm. Using the formula, that gives me an appropriate scale length of

$$\text{scale length} = 98.8 \times \ln(205) + 115.3 = 641.2 \text{ mm}$$

So if I was building a guitar for me to play, I would choose a scale length of 640 mm. According to this model, that is! Interestingly, that's right between the standard scale lengths used by Fender and Gibson (see Appendices B and C).

It's interesting, though, that I have often thought that it is easier for me to play smaller guitars than "standard" ones. So a lot of this is all down to *feel*. You really need to try guitars of different sizes out, and see which are most comfortable for you to play.

6 The Problem With Theoretical Fret Positioning

So now we've got a theoretical method for positioning frets, and a formula to go with it, so that we can place frets perfectly. But sometimes, when I play a guitar, strings fretted near the nut produce slightly sharp notes. Why is this?

Well, to cut a long story short, the reason is that when you fret a string, you have to use force to bend the string down onto the fret. That changes things.

When you fret a string, you are increasing the length of the string slightly. And that's going to affect our ideal fret positioning equation (3), because it affects the length of the string that's vibrating.

Some couple of papers that I've read recently that try to address these issues are Smoyer (2005), Lane and Kasparis (2012) and Varieschi and Gower (2010). These are quite complicated papers to understand, so what I've done is to come up with my own simplified version of their work. My mathematical model that resulted is described in detail in Smith (2014).

6.1 Theory and Practice

The problems associated with the Western Scale (see Section 3) mean that if you use Equation (3) to place your frets, there will be errors in frequencies on most frets, no matter how you tune your guitar. It is just to be hoped that the errors are too small for the normal human ear to notice.

It is possible to come up with fret positions for any fret of any string that would play a given note perfectly. Unfortunately, that would result in a fretboard that was incredibly difficult to make, and would also be rather difficult to play. Bending strings, for example, would be rather interesting. See, for example, True Temperament (2014).

Instead, luthiers (guitar makers) have come up with a neat idea for helping to reduce the frequency errors at frets. It's called *intonation*, and it involves the use of a bridge whose position for each string can be varied. Using a variable-position bridge, it is possible to reduce the errors in frequency when playing particular frets. Variable-position bridges have been around for a very long time, and they do help. But they don't get rid of all fret errors.

So it occurred to me that it would be even better if we had a *variable-position nut* as well. This gives us another variable to play with when intonating the guitar. Fender, Gibson, and almost all other guitar maufacturers have fixed nuts. Also, most guitar manufacturers have nuts that position the nut in *exactly the same place for each string*.

So when I came up with this variable-nut idea, I saw the £-signs lighting up before my eyes. Not even Helmuth Lemme, in his bible on the subject of electric guitars, (Lemme (2012)), addresses the possibility of an adjustable nut.

It didn't take very long, however, to discover that I am not the first to come up with it. In doing a bit of research on variable-position nuts, I came across the following, for example: Hartley (2014), and Earvana (2014). So alas, I am not going to be rich and famous. But, there doesn't seem to be very much research into the placement of nuts and bridges on a guitar string. Hence my mathematical model (Smith (2014)).

7 Conclusion

The upshot of all this is that:

- when making a guitar, place the frets according to Equation (3), for each string;
- ensure the bridge position is variable for each string;
- ensure the nut position is variable for each string.

Then, use intonation via the adjustable nut and the adjustable bridge for each of the strings so that the error in frequency of any note is small enough.

To give you a start in knowing where to place the nut and bridge for a given string, and an algorithm for intonating the guitar, see (Smith (2014)).

A Alternative Fret Position Formula

Starting with

$$L_k = \frac{S}{r^k}$$

we could massage this about a bit:

$$\begin{aligned}
L_k &= \frac{S}{r^k} \\
&= \frac{S}{2^{\frac{k}{12}}} \\
&= 2^{-\frac{k}{12}} \times S \\
&= 2^{-\frac{k}{12}} \times 2 \times \left(\frac{S}{2}\right) \\
&= 2^{-\frac{k}{12}+1} \times \left(\frac{S}{2}\right) \\
&= 2^{\frac{-k+12}{12}} \times \left(\frac{S}{2}\right) \\
&= 2^{\frac{12-k}{12}} \times \left(\frac{S}{2}\right)
\end{aligned}$$

Now if we let $n = 12 - k$, so that n is another way of counting frets, but starting with $n = 12$ at the nut, $n = 11$ at the first fret, $n = 10$ at the seconds fret, ..., $n = 0$ at the 12^{th} fret, $n = -1$ at the 13^{th} fret, etc, then

$$L_k = 2^{\frac{n}{12}} \left(\frac{S}{2}\right)$$

and

$$\begin{aligned}
F_k &= S - 2^{\frac{n}{12}} \left(\frac{S}{2}\right) \\
&= 2\left(\frac{S}{2}\right) - 2^{\frac{n}{12}} \left(\frac{S}{2}\right) \\
&= 2\left(\frac{S}{2}\right) - 2^{\frac{n}{12}} \left(\frac{S}{2}\right) \\
&= \left(2 - 2^{\frac{n}{12}}\right) \left(\frac{S}{2}\right)
\end{aligned}$$

B Fret Positions for a Fender Stratocaster

A Fender Stratocaster has a scale length of 25.5 inches (647.7 mm). Using the formula given by Equation (3), fret positions are:

Table 2: Fret Positions for a 25.5 inch (647.7 mm) Scale Length

Fret Number	Fret Position From Nut (inches)	Fret Position From Nut (mm)
0	0.0000	0.000
1	1.4312	36.353
2	2.7821	70.665
3	4.0571	103.051
4	5.2606	133.620
5	6.3966	162.473
6	7.4688	189.707
7	8.4808	215.412
8	9.4360	239.675
9	10.3376	262.575
10	11.1886	284.191
11	11.9918	304.593
12	12.7500	323.850
13	13.4656	342.026
14	14.1410	359.182
15	14.7786	375.376
16	15.3803	390.660
17	15.9483	405.087
18	16.4844	418.703
19	16.9904	431.556
20	17.4680	443.687
21	17.9188	455.138
22	18.3443	465.945
23	18.7459	476.146
24	19.1250	485.775

C Fret Positions for a Gibson Les Paul

A Gibson Les Paul has a scale length of 24.75 inches (628.65 mm). Using the formula given by Equation (3), fret positions are:

Table 3: Fret Positions for a 24.75 inch (628.65 mm) Scale Length

Fret Number	Fret Position From Nut (inches)	Fret Position From Nut (mm)
0	0.0000	0.000
1	1.3891	35.283
2	2.7003	68.587
3	3.9378	100.020
4	5.1059	129.690
5	6.2084	157.695
6	7.2491	184.127
7	8.2314	209.076
8	9.1585	232.625
9	10.0336	254.852
10	10.8595	275.832
11	11.6391	295.634
12	12.3750	314.325
13	13.0696	331.967
14	13.7251	348.618
15	14.3439	364.335
16	14.9280	379.170
17	15.4792	393.172
18	15.9996	406.389
19	16.4907	418.863
20	16.9542	430.638
21	17.3918	441.751
22	17.8048	452.241
23	18.1946	462.142
24	18.5625	471.488

References

Earvana, L. L. C. (2014). Earvana : Compensated tuning systems. http://www.earvana.com/.

Harkleroad, L. (2006). *The Math Behind the Music*. Cambridge University Press.

Hartley, P. (2014). Phil Hartley Guitar Repairs: The Funky Nut. http://www.guitarsetup.co.uk/the-funky-nut.php.

kechance et al. (2012). Hand Size and Scale Length. A thread published on the Classical Guitar classicalguitardelcamp forum. http://www.classicalguitardelcamp.com/viewtopic.php?f=71&t=53237&start=30.

Lane, J. and Kasparis, T. (2012). A Frequency Error Model for Fretted String Instruments. *Acta Acustica united with Acustica* **98**, 936–944.

Lemme, H. (2012). *Electric Guitar: Sound Secrets and Technology*. Elektor International Media, first edition.

Nelkon, M. and Parker, P. (1988). *Advanced Level Physics*. Heinemann, sixth edition.

Smith, S. (2014). How to Intonate a Guitar.

Smoyer, L. (2005). *Musical Mathematics, The Mathematical Structure of the Pythagorean and Equal Tempered Scale*. Master's thesis, Portland State University, Department of Mathematics and Statistics.

True Temperament (2014). True temperament AB. http://www.truetemperament.com.

Varieschi, G. and Gower, C. (2010). Intonation and Ccompensation of Fretted String Instruments. *American Journal of Physics* **78**, 47–55.