Steve Smith Tuition: Physics Notes

Static Fields IV: Gravity Examples

Contents

Prerequisites

You should really have read my other documents on gravity: Smith (2014a) and Smith (2014b).

You should also know the equations relating to circular motion.

Notes

None.

Document History

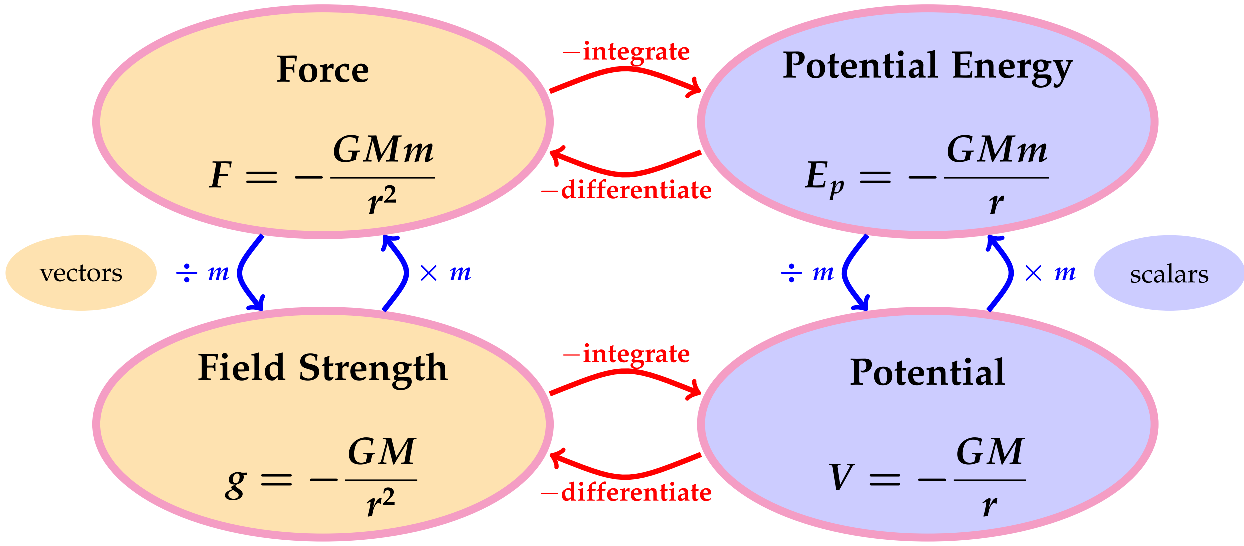
Date	Version	Comments
20th December 2014	1.0	Initial creation of the document.

1 Gravitational Field Equations

As a quick reminder, here are the pictures that you should learn to master gravitational fields. You should know what all the symbols mean by now!

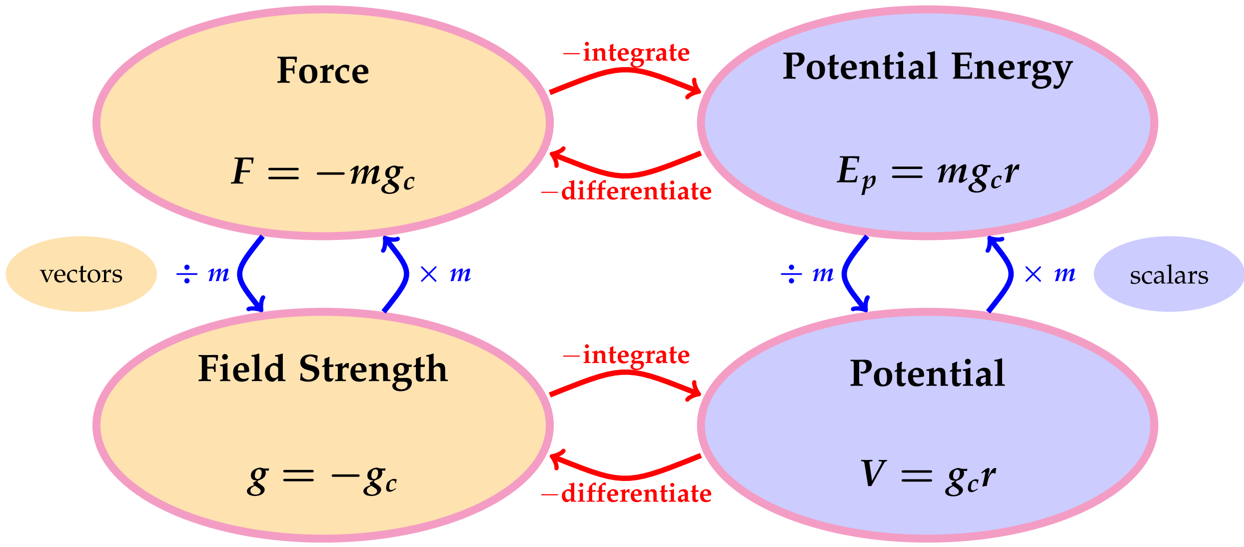
1.1 Radial Gravitational Field Equations

Figure 1: Obtaining Radial Gravity Equations IV: The Potential



1.2 Uniform Gravitational Field Equations

Figure 2: Obtaining Uniform Gravity Equations IV: The Potential



2 The Questions

Except where stated, use the values of G and r_E (the radius of the Earth) from AppendixA.

2.1 Example 1

Calculate the force of attraction between two small objects of mass 5 and 8 kg respectively, which are a distance of 10 cm apart.

2.2 Example 2

If the acceleration due to gravity is 9.8 m s^{-2} at the surface of the Earth, calculate a value for the mass of the Earth. Give the theory.

[Assume that you only know the radius of the Earth and the gravitational constant, G.]

2.3 Example 3

Assuming that the mean density of the earth is 5500 kg m^{-3}, find a value for the acceleration due to gravity at the Earth's surface. Derive the formula used.

[Assume that you only know the radius of the Earth and the gravitational constant, G.]

2.4 Example 4

The gravitational force on a mass of 1 kg at the Earth's surface is 10 N. Assuming the Earth is a sphere of radius R, calculate the gravitational force on a satellite of mass 100 kg in a circular orbit of radius $2R$ from the centre of the Earth.

[Assume that you DO NOT know the radius or the mass of the Earth, nor the gravitational constant, G.]

2.5 Example 5

Figure 3: Equipotentials

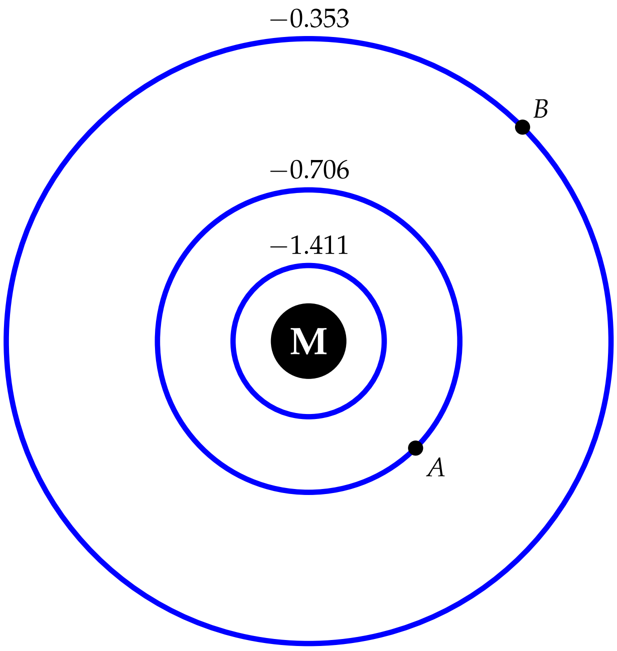


Figure 3 shows some equipotential lines of the gravitational field around a large mass M. (It actually represents the Moon.) The values shown are in units of $MJ\ kg^{-1}$.

(a) What are equipotential lines?

(b) How much energy would it take to move a satellite of mass 250 kg from A to B in this field?

2.6 Example 6

(a) State Newton's Law of gravitation.

(b) Starting from Newton's Law of gravitation, and assuming the Earth is a uniform sphere of mass M and radius R, show that the acceleration of free fall at the Earth's surface is

$$g = -\frac{GM}{R^2}$$

(c) What is the acceleration of a satellite moving in a circular orbit of radius $2R$ around the Earth?

[Assume that you know the radius and mass of the Earth and the gravitational constant, G.]

2.7 Example 7

(a) A satellite X moves around the earth in a circular orbit of radius R. Another satellite Y of the same mass moves around the earth in a circular orbit of radius $4R$. Show that:

- (i) the speed of X is twice that of Y;
- (ii) the kinetic energy of X is greater than that of Y;
- (iii) the potential energy of X is less than that of Y.

(b) Has X or Y the greater total energy (kinetic plus potential energy)?

2.8 Example 8

Find the period of revolution (the time for one complete orbit) of a satellite moving in a circular orbit around the Earth at a height of 3.6×10^6 m above the Earth's surface.

[Assume that you know the radius and mass of the Earth, and the gravitational constant, G.]

2.9 Example 9

(a) Assuming that the planets are moving in circular orbits, apply Kepler's laws to show that the acceleration of a planet is inversely proportional to the square of its distance from the sun. Explain the significance of this and show clearly how it leads to Newton's law of universal gravitation.

2.10 Example 10

A proposed communication satellite would revolve round the Earth in a circular orbit in the equatorial plane, at a height of 35880 km above the Earth's surface. Find the period of revolution of the satellite in hours, and comment on the result.

[Assume that you know the radius and mass of the Earth, and the gravitational constant, G.]

2.11 Example 11

Calculate the gravitational potential energy of the Earth-Moon system.

[Assume that you know the masses of the Earth and the Moon, the eart-Moon distance, and the gravitational constant, G.]

2.12 Example 12

What is the potential energy of a 5 kg mass at a height of 20 m above the ground, assuming that the Earth's gravitational field is uniform up to that height?

[Assume that you know the gravitational field strength at the surface of the Earth.]

2.13 Example 13

(a) The gravitational potential at the surface of the Earth is -6.26×10^7 J kg^{-1}. What does this mean?

(b) What the the minimum initial velocity that a spacecraft would have to have to be able to escape from the Earth's gravitational field?

2.14 Example 14

How much energy would it take to move a satellite of mass 250 kg from a circular orbit with a radius of 10000 km to a circular orbit of radius 12000 km?

[Assume that you know the mass of the Earth, and the gravitational constant, G.]

3 The Answers

3.1 Example 1

Calculate the force of attraction between two small objects of mass 5 and 8 kg respectively, which are a distance of 10 cm apart.

From Figure 1, the force between two objects due to gravity is

$$F = -\frac{GMm}{r^2}$$

There is no need to worry about the sign of this equation, as we know that the force is attractive. So, ignoring the sign, the size of the force between the masses will be

$$F = \frac{6.67 \times 10^{-11} \cdot 5 \cdot 8}{(10 \times 10^{-2})^2} = 2.67 \times 10^{-7}\ N \text{ (to 3 sf)}$$

3.2 Example 2

If the acceleration due to gravity is 9.8 m s^{-2} at the surface of the Earth, calculate a value for the mass of the Earth. Give the theory.

[Assume that you only know the radius of the Earth and the gravitational constant, G.]

From Figure 1, the force between two objects due to gravity is

$$F = -\frac{GMm}{r^2}$$

There is no need to worry about the sign of this equation, as we know that the force is attractive. So, ignoring the sign, the size of the force between the masses will be

$$F = \frac{GMm}{r^2}$$

Now also from Figure 1, the relationship between the force of gravity and the gravitational field strength (which is the same thing as the acceleration due to gravity) is

$$g = \frac{F}{m}$$

and so

$$g = \frac{GMm}{r^2 m} = \frac{GM}{r^2}$$

If we make M the subject of this equation,

$$M = \frac{gr^2}{G}$$

then, plugging the numbers in (from Section A), we get

$$M = \frac{9.8 \cdot (6.371 \times 10^6)^2}{6.67 \times 10^{-11}} = 5.96 \times 10^{24}\ kg \text{ (to 3 sf)}$$

3.3 Example 3

Assuming that the mean density of the earth is 5500 kg m^{-3}, find a value for the acceleration due to gravity at the Earth's surface. Derive the formula used.

[Assume that you only know the radius of the Earth and the gravitational constant, G.]

The definition of density ρ in terms of mass m and volume V is

$$\rho = \frac{m}{V}$$

So if we make m the subject of this equation

$$m = \rho V$$

Now assuming that Earth is a sphere of radius R, then its volume will be

$$V = \tfrac{4}{3}\pi R^3$$

and so the mass of the Earth will be

$$m = \tfrac{4}{3}\pi \rho R^3 \qquad (1)$$

From Figure 1, the force between two objects due to gravity is

$$F = -\frac{GMm}{r^2}$$

There is no need to worry about the sign of this equation, as we know that the force is attractive. So, ignoring the sign, the size of the force between the masses will be

$$F = \frac{GMm}{r^2}$$

Now also from Figure 1, the relationship between the force of gravity and the gravitational field strength (which is the same thing as the acceleration due to gravity) is

$$g = \frac{F}{m}$$

and so

$$g = \frac{GMm}{r^2 m} = \frac{GM}{r^2}$$

Right, so now we can insert the mass of the Earth we found in Equation (1) into this equation:

$$g = \frac{4\pi G \rho R^3}{3R^2}$$

and, cancelling,

$$g = \tfrac{4}{3}\pi G \rho R$$

Plugging the numbers in:

$$g = \tfrac{4}{3}\pi \cdot 6.67 \times 10^{-11} \cdot 5500 \cdot 6.371 \times 10^{6} = 9.79\ ms^{-2} \text{ (to 3 sf)}$$

3.4 Example 4

The gravitational force on a mass of 1 kg at the Earth's surface is 10 N. Assuming the Earth is a sphere of radius R, calculate the gravitational force on a satellite of mass 100 kg in a circular orbit of radius $2R$ from the centre of the Earth.

[Assume that you DO NOT know the radius or the mass of the Earth, nor the gravitational constant, G.]

From Figure 1, the force between two objects due to gravity is

$$F = -\frac{GMm}{r^2}$$

where F is the force on a mass m a distance r away from a large mass M. G is the gravitational constant. The $-$ sign shows that the force is attractive.

In this question, the signs (directions of forces) are irrelevant, so I will ignore them.

So, if the force on a 1 kg mass a distance R from a large mass M is 10 N, then,

$$10 = \frac{GM \cdot 1}{R^2}$$

or

$$GM = 10R^2$$

So the force on an object of mass 100 kg a distance of $2R$ from the same large mass M would be

$$F = \frac{GMm}{r^2} = \frac{10R^2 \cdot 100}{(2R)^2}$$

and, simplifying a bit,

$$F = \frac{1000R^2}{4R^2} = 250\ N$$

3.5 Example 5

Figure 4: Equipotentials

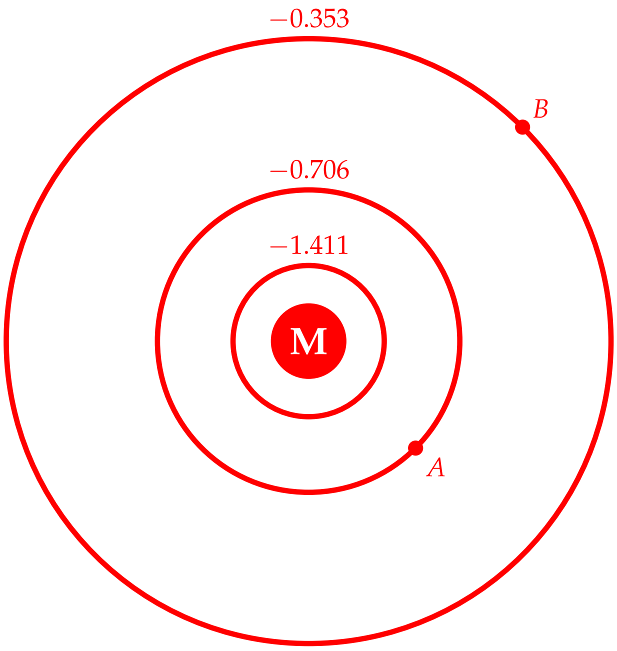


Figure 4 shows some equipotential lines of the gravitational field around a large mass M. (It actually represents the Moon.) The values shown are in units of $MJ\ kg^{-1}$.

(a) What are equipotential lines?

(b) How much energy would it take to move a satellite of mass 250 kg from A to B in this field?

(a) An equipotential line represents points in the field where the potential is the same. They're like the contour lines on a map.

(c) Since this is a radial gravitational field, from Figure 1, the relationship between potential V and potential energy E_p is

$$E_p = Vm$$

So to find the potential energy difference between A and B we find the potential difference, and multiply by m, the mass of the satellite:

$$E_p = (\text{potential at B} - \text{potential at A}) \times m$$

so

$$E_p = (-0.353 \times 10^6 - (-0.706 \times 10^6)) \cdot 250 = 8.83 \times 10^7\ J \text{ (to 3 sf)}$$

3.6 Example 6

(a) State Newton's Law of gravitation.

(b) Starting from Newton's Law of gravitation, and assuming the Earth is a uniform sphere of mass M and radius R, show that the acceleration of free fall at the Earth's surface is

$$g = -\frac{GM}{R^2}$$

(c) What is the acceleration of a satellite moving in a circular orbit of radius $2R$ around the Earth?

[Assume that you know the radius and mass of the Earth and the gravitational constant, G.]

(a) From Figure 1, the force between two objects due to gravity, passed down to us by Newton, is

$$F = -\frac{GMm}{R^2}$$

(b) Now also from Figure 1, the relationship between the force of gravity and the gravitational field strength (which is the same thing as the acceleration due to gravity) is

$$g = \frac{F}{m}$$

and so

$$g = -\frac{GMm}{R^2 m} = -\frac{GM}{R^2}$$

(c) The acceleration of an object moving in a circular orbit of radius $2R$ around the Earth will be

$$g = -\frac{GM}{(2R)^2} = -\frac{GM}{4R^2}$$

So, using the data from Section A, this will be

$$g = -\frac{6.67 \times 10^{-11} \cdot 5.972 \times 10^{24}}{4 \cdot (6.371 \times 10^6)^2} = 2.45\ ms^{-2} \text{ (to 3 sf)}$$

3.7 Example 7

(a) A satellite X moves around the earth in a circular orbit of radius R. Another satellite Y of the same mass moves around the earth in a circular orbit of radius $4R$. Show that:

- (i) the speed of X is twice that of Y;
- (ii) the kinetic energy of X is greater than that of Y;
- (iii) the potential energy of X is less than that of Y.

(b) Has X or Y the greater total energy (kinetic plus potential energy)?

(a) (i) From Figure 1, the force between two objects due to gravity is

$$F = -\frac{GMm}{r^2}$$

There is no need to worry about the sign of this equation, as we know that the force is attractive. So, ignoring the sign, the size of the gravitational force F_g between the satellite X and the Earth will be

$$F_g = \frac{GMm}{R^2}$$

where M is the mass of the Earth and m the mass of the satellite in orbit around the Earth with radius R.

Now, the centripteal force F_c necessary for the satellite X to go around the Earth in a circular orbit of radius R with speed v_X is

$$F_c = \frac{mv_X^2}{R}$$

For the satellite X, the gravitational attraction of the Earth is providing that centripetal force, so

$$F_c = F_g$$

and so

$$\frac{mv_X^2}{R} = \frac{GMm}{R^2}$$

Doing a bit of simplifying, this boils down to

$$v_X^2 = \frac{GM}{R} \tag{2}$$

Now the same relationship must hold for satellite Y. But Y has an orbital radius of $4R$. So for Y,

$$v_Y^2 = \frac{GM}{4R} \tag{3}$$

Putting equations (2) and (3) together, we get

$$\left(\frac{GM}{R} =\right) v_X^2 = 4v_Y^2$$

and so

$$v_X = 2v_Y$$

(ii) Well the kinetic energy of an object is $\frac{1}{2}mv^2$, so if the two satellites have the same mass, but the speed of X is greater than (in fact twice) that of stalleite Y, then satellite X will have more kinetic energy (two^2 times more) than satellite Y.

(iii) From Figure 1, the potential energy of an object of mass m in a circular orbit of radius r about another mass M is

$$E_p = -\frac{GMm}{r}$$

So the potential energy of X will be

$$E_{pX} = -\frac{GMm}{R}$$

and the potential energy of Y will be

$$E_{pY} = -\frac{GMm}{4R}$$

Since

$$-\frac{GMm}{4R} > -\frac{GMm}{R}$$

(this time the sign is very important!), then the potential energy of Y is greater than the potential energy of X.

(b) The total energy of X will be

$$\text{total energy, } E_{TX} = \text{kinetic energy, } E_k + \text{potential energy, } E_p$$

So

$$E_{TX} = \tfrac{1}{2}mv_X^2 + \left(-\frac{GMm}{R}\right)$$

or

$$E_{TX} = \tfrac{1}{2}m\frac{GM}{R} - \frac{GMm}{R} = -\frac{GMm}{2R}$$

using Equation (2).

(b) The total energy of Y will be

$$\text{total energy, } E_{TY} = \text{kinetic energy, } E_k + \text{potential energy, } E_p$$

So

$$E_{TY} = \tfrac{1}{2}mv_Y^2 + \left(-\frac{GMm}{4R}\right)$$

or

$$E_{TY} = \tfrac{1}{2}m\frac{GM}{4R} - \frac{GMm}{R} = -\frac{GMm}{8R}$$

using Equation (3). So satellite Y has the greater total energy, since

$$-\frac{GMm}{8R} > -\frac{GMm}{2R}$$

the minus sign being crucial.

3.8 Example 8

Find the period of revolution (the time for one complete orbit) of a satellite moving in a circular orbit around the Earth at a height of 3.6×10^6 m above the Earth's surface.

[Assume that you know the radius and mass of the Earth, and the gravitational constant, G.]

(a) (i) From Figure 1, the force between two objects due to gravity is

$$F = -\frac{GMm}{r^2}$$

There is no need to worry about the sign of this equation, as we know that the force is attractive. So, ignoring the sign, the size of the gravitational force F_g between the satellite and the Earth will be

$$F_g = \frac{GMm}{R^2}$$

where M is the mass of the Earth and m the mass of the satellite in orbit around the Earth with radius R.

Now, the centripteal force F_c necessary for the satellite to go around the Earth in a circular orbit of radius R with speed v is

$$F_c = \frac{mv^2}{R}$$

For the satellite, the gravitational attraction of the Earth is providing that centripetal force, so

$$F_c = F_g$$

and so

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

Doing a bit of simplifying, this boils down to

$$v^2 = \frac{GM}{R} \tag{4}$$

Since the orbit of the satellite is circular, then the distance d travelled in one revolution will be

$$d = 2\pi R$$

and since the definition of speed is

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

then the period of the orbit T will ve given by

$$T = \frac{d}{v}$$

So,

$$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}$$

Now the radius of this satellite's orbit will be $6.371 \times 10^6 + 3.6 \times 10^6 = 9.971 \times 10^6$ m, so, plugging the numbers in:

$$T = \frac{2\pi \cdot 9.971 \times 10^6}{\sqrt{\frac{6.67\times10^{-11}\cdot5.972\times10^{24}}{9.971\times10^6}}} = 9910 \ s \text{ (to 3 sf)}$$

which is about 2 hours, 45 minutes.

3.9 Example 9

(a) Assuming that the planets are moving in circular orbits, apply Kepler's laws to show that the acceleration of a planet is inversely proportional to the square of its distance from the sun. Explain the significance of this and show clearly how it leads to Newton's law of universal gravitation.

(a) The centripetal acceleration a_c necessary for an object of mass m to move in a circular orbit of radius R with speed v is

$$a_c = \frac{v^2}{R}$$

Kepler had discovered that the square of the period T of a planet in our solar system was proportional to the cube of its average distance R from the Sun:

$$T^2 = kR^3$$

k being the constant of proprtionality. So if a planet is in a circular orbit of radius R, then it will travel a distance d of

$$d = 2\pi R$$

in each orbit. But distance d, speed v and time T are related by

$$v = \frac{d}{T}$$

so

$$T = \frac{d}{v} = \frac{2\pi R}{v}$$

Squaring both sides

$$T^2 = \frac{4\pi^2 R^2}{v^2}$$

So

$$v^2 = \frac{4\pi^2 R^2}{T^2}$$

And using Kepler's Law,

$$v^2 = \frac{4\pi^2 R^2}{kR^3} = \frac{4\pi^2}{kR}$$

Plugging this into the centripetal acceleration equation:

$$a_c = \frac{\frac{4\pi^2}{kR}}{R} = \frac{K}{R^2}$$

where $K = \frac{4\pi}{k}$. So the acceleration needed to keep a planet in motion around the Sun must be inversely proportional to the square of the distance.

From Newton's Second Law, $F = ma$, the force F necessary to keep the planet (of mass m) in a circular orbit of radius R will be

$$F = \frac{K}{R^2} \cdot m = \frac{Km}{R^2} \quad (5)$$

But by Newton's Third Law, if the Sun exerts this force on the planet, then the planet must exert this force on the Sun! Looking at the problem the other way around, thinking that the Sun must be going around in a circular orbit due to the force of the planet on it, then there must be an equivalent equation to Equation (5):

$$F = \frac{K'M}{R^2} \quad (6)$$

where M is the mass of the Sun.

As this same force F has to be proportional to both m and M, then both of these equations can only be true if

$$F = \frac{GMm}{r^2}$$

incorporating both masses into the equation.

3.10 Example 10

A proposed communication satellite would revolve round the Earth in a circular orbit in the equatorial plane, at a height of 35880 km above the Earth's surface. Find the period of revolution of the satellite in hours, and comment on the result.

[Assume that you know the radius and mass of the Earth, and the gravitational constant, G.]

(a) (i) From Figure 1, the force between two objects due to gravity is

$$F = -\frac{GMm}{r^2}$$

There is no need to worry about the sign of this equation, as we know that the force is attractive. So, ignoring the sign, the size of the gravitational force F_g between the satellite and the Earth will be

$$F_g = \frac{GMm}{R^2}$$

where M is the mass of the Earth and m the mass of the satellite in orbit around the Earth with radius R.

Now, the centripteal force F_c necessary for the satellite to go around the Earth in a circular orbit of radius R with speed v is

$$F_c = \frac{mv^2}{R}$$

For the satellite, the gravitational attraction of the Earth is providing that centripetal force, so

$$F_c = F_g$$

and so

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

And simplifying a bit we get

$$v^2 = \frac{GM}{R}$$

But, in a circular orbit of radius R, the distance d the satellite travels in one revolution will be

$$d = 2\pi R$$

Now since distance d, speed v and time T are related by

$$v = \frac{d}{T} = \frac{2\pi R}{T}$$

and so

$$v^2 = \frac{4\pi^2 R^2}{T^2}$$

then

$$\frac{GM}{R} = \frac{4\pi^2 R^2}{T^2}$$

So,

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

and

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}}$$

Plugging the numbers in:

$$T = \sqrt{\frac{4\pi^2 (6.371 \times 10^6 + 35880 \times 10^3)^3}{6.67 \times 10^{-11} \cdot 5.972 \times 10^{24}}} = 86459\ s$$

which is almost exactly 24 hrs.

35880 km will be the height a satellite needs to orbit the Earth in order for the satellite's period to be exactly the same as that of the Earth. Consequently, if placed in an orbit over the equator, the satellite will remain over the same point on the Earth.

3.11 Example 11

Calculate the gravitational potential energy of the Earth-Moon system.

[Assume that you know the masses of the Earth and the Moon, the eart-Moon distance, and the gravitational constant, G.]

From Figure 1, the potential energy between two objects due to gravity is

$$E_p = -\frac{GMm}{r}$$

Plugging the numbers in:

$$E_p = -\frac{6.67 \times 10^{-11} \cdot 5.972 \times 10^{24} \cdot 7.347 \times 10^{22}}{3.844 \times 10^{8}} = -7.61 \times 10^{28}\ J \text{ (to 3 sf)}$$

3.12 Example 12

What is the potential energy of a 5 kg mass at a height of 20 m above the ground, assuming that the Earth's gravitational field is uniform up to that height?

[Assume that you know the gravitational field strength at the surface of the Earth.]

From Figure 2, the potential energy of an object due to gravity in a uniform field is

$$E_p = mgr$$

Plugging the numbers in:

$$E_p = 5 \cdot 9.81 \cdot 20 = 981\ J$$

3.13 Example 13

(a) The gravitational potential at the surface of the Earth is -6.26×10^7 J kg^{-1}. What does this mean?

(b) What the the minimum initial velocity that a spacecraft would have to have to be able to escape from the Earth's gravitational field?

(a) From Figure 1, gravitational potential is the gravitational potential energy per unit mass at a particular point in the gravitational field. The convention is for a point infinitely far away from the center of the gravitational field to have zero potential.

To get zero potential from -6.26×10^7 J kg^{-1}, we would have to add 6.26×10^7 J kg^{-1}. So 6.26×10^7 J is the energy *per kilogram* that we would need to move an object from the surface of the Earth to infinity. That is, this is the amount of energy we would need to supply per kilogram for an object to escape the gravitational pull of the Earth.

(b) Let's say that we had an object that had a mass m. Then for it to escape the gravitational pull of the Earth, we would need to give it $6.26 \times 10^7 \times m$ J of energy. How can we supply this energy? If we were to give the object an initial kinetic energy of this amount, then that would do it, since as the object moved away from the Earth, it would gain the potential energy it needs to escape by losing its kinetic energy (so that its total energy remains constant). So:

$$\tfrac{1}{2}mv^2 = 6.26 \times 10^7 \times m$$

Cancelling the ms and simplifying:

$$v = \sqrt{2 \times 6.26 \times 10^7} = 1.12 \times 10^4\ ms^{-1} = 11.2\ kms^{-1}$$

3.14 Example 14

How much energy would it take to move a satellite of mass 250 kg from a circular orbit with a radius of 10000 km to a circular orbit of radius 12000 km?

[Assume that you know the mass of the Earth, and the gravitational constant, G.]

From Figure 1, the potential energy of an object due to gravity is

$$E_p = -\frac{GMm}{r}$$

So we could find the potential energies of the object at the two points and find the difference:

$$E_{p10000} = -\frac{6.67 \times 10^{-11} \cdot 5.972 \times 10^{24} \cdot 250}{10000 \times 10^3} = -9.9583 \times 10^9 \ J \text{ (to 5 sf)}$$

And

$$E_{p12000} = -\frac{6.67 \times 10^{-11} \cdot 5.972 \times 10^{24} \cdot 250}{12000 \times 10^3} = -8.2986 \times 10^9 \ J \text{ (to 5 sf)}$$

So the difference is

$$E_{p12000} - E_{p10000} = -8.30 \times 10^9 - (-9.96 \times 10^9) = 1.66 \times 10^9 \ J \text{ (to 3 sf)}$$

Now you could do this another way. Instead of finding the potential energy at each point and finding the difference, we could work out the potential at each point, then find the potential difference, and multiply that by the mass.

4 Appendices

A Useful Data

Quantity	Value
Earth - Moon Distance	3.844×10^{8} m
Gravitational Constant	6.673×10^{-11} m^{3} kg^{-1} s^{-2}
Mass of the Earth	5.972×10^{24} kg
Radius of the Earth	6.371×10^{6} m
Mass of the Moon	7.347×10^{22} kg
Gravitational Field Strength at the surface of the Earth	9.81 ms^{-2}
k	8.9876×10^{9} m^{3} kg s^{-4} A^{-2}
ϵ	8.8542×10^{-12} m^{-3} kg^{-1} s^{4} A^{2}

References

Smith, S. (2014a). Static Fields I: Gravity. An introduction to fields through Newton's Law of Gravity.

Smith, S. (2014b). Static Fields II: Gravity Essentials. How to obtain all the gravitational field equations.