Steve Smith Tuition: Physics Notes

Static Fields II: Gravity Essentials

Contents

Prerequisites

I'm assuming here that you have had the stamina to read through my document Smith (2014a). This explains the relationships between all the key field concepts, and where the equations come from.

If you haven't read that document, you probably should, but if you want to cut to the chase, and you want the short route to answering fields questions, then so long as you are prepared to take a few things on trust, this is the document for you!

Notes

None.

Document History

Date	Version	Comments
16th October 2014	1.0	Initial creation of the document.
28th March 2015	1.1	Adding a note to explain why there is no constant of integration in the potential energy and potential calculations in the case of a uniform gravitational field.

1 Introduction

Here, we are only interested in static fields. That is, fields that don't change with time. There are three static fields that A-Level students encounter: gravitational fields, electric fields and magnetic fields. This document is only concerned with gravitational fields. For a discussion of electric fields see Ref Smith (2014c), and to find out about magnetic fields, see Ref Smith (2015).

For a document that summarises gravitational fields, see Smith (2014b); for some example questions on gravitational fields, see Smith (2014d); for some example questions on electric fields, see Smith (2014e).

1.1 What is a Field?

In physics, a field is simply a region of space where an object of a particular kind would experience a force. What I mean by *a particular kind* is that objects experience forces because of some property that they have. For example, an object that has *mass* experiences a *gravitational* force when located in a *gravitational* field. And an object that has *charge* experiences an *electric* force when located in an *electric* field. Interestingly, gravitational fields are caused by masses, and electric fields are caused (at A-level, at least) by electric charges.

1.2 Radial and Uniform Fields

There are two kinds of gravitational or electric fields that you come across at A-Level. They are known as *radial* and $uniform$ fields.

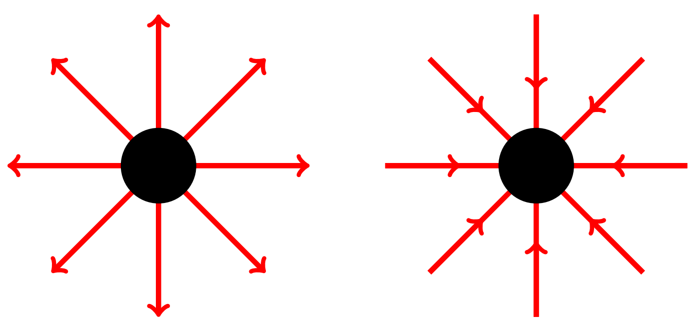
1.2.1 Radial Fields

Radial fields look like those in Figure 1. Radial fields are caused by central objects, often considered to be points. Examples of radial fields include:

- gravitational fields caused by a large central mass, such as a star or a planet;
- electric fields caused by a central charge, such as a proton or an electron.

We call these radial fields because the field lines (indicated in red in Figure 1) look like radiuses of spheres centered on the point objects.

Figure 1: Radial Fields



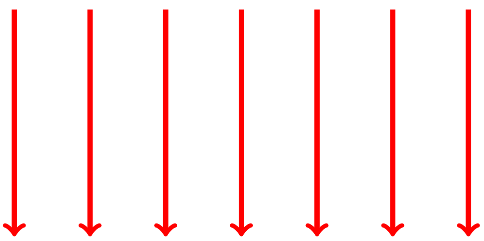
1.2.2 Uniform Fields

Uniform fields look like that in Figure 2. Uniform fields are caused by extended objects, often planes. Examples of uniform fields include:

- gravitational fields, where the scale of the situation is very small compared with the size of the object creating the field;
- electric fields caused by capacitor plates.

We call these uniform fields because the field lines (indicated in red in Figure 2) all point in the same direction, and are uniformly distributed in space (another way of saying that the field strength (see later) is the same everywhere).

Figure 2: Uniform Fields

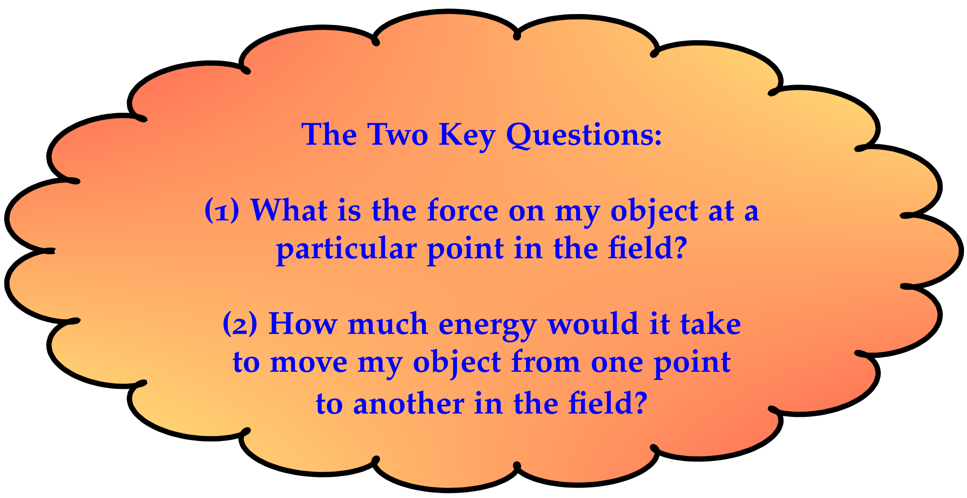


1.3 The Two Key Questions

When analysing fields, there are only two questions that we need to answer. These are:

- **What is the force on my object when it is placed at a particular point in the field?**
- **How much energy will it take to move my object from one point to another in the field?**

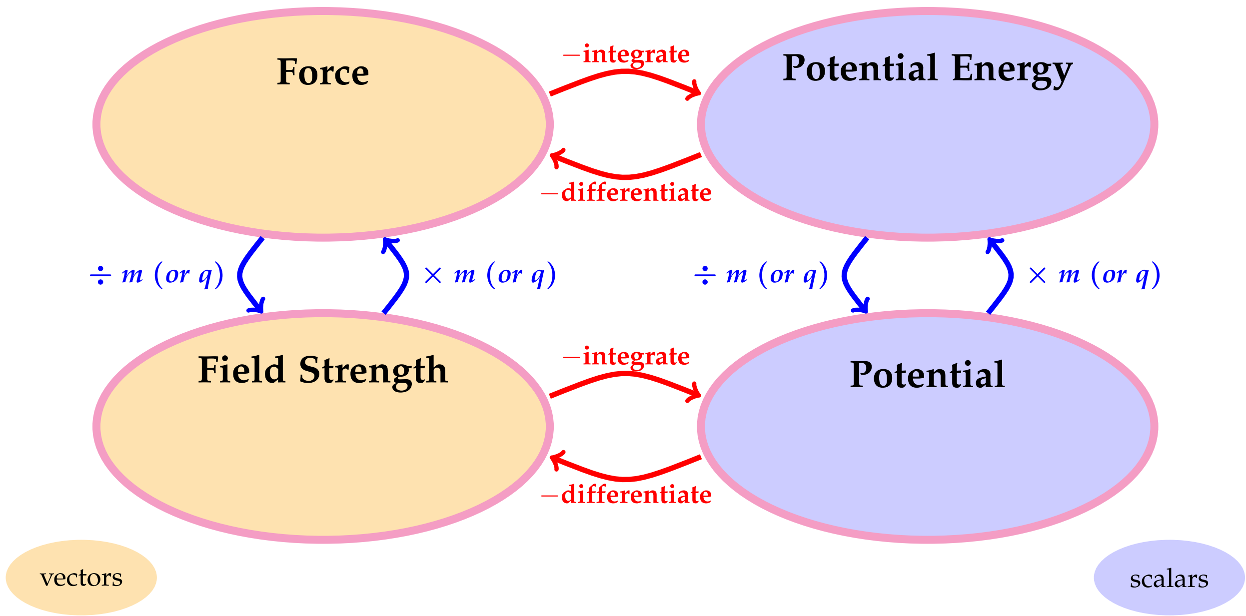
When you study anything to do with fields, these are the two things to have uppermost in your mind as you read. Keep asking yourself these two questions. Everything about fields boils down to these two key questions. As you read on, don't forget to keep these questions in mind. Have I emphasised that enough? You know, I'm not sure I have! What were those questions again...?



2 The Relationship Between the Key Concepts

In (Smith, 2014a), I presented the picture representing the four key concepts for gravitational fields. Here is the same diagram, but this time I've removed the equations so the concepts themselves stand out.

Figure 3: Summary of Fields



The reason for the m (*or* q) business is that the thing you have to multiply or divide by depends on the type of field you have. Gravitational fields affect masses, so you would use m if you were looking at a gravitational field; electric fields affect charges, so you would use q if you were looking at an electric field.

Because these relationships are the same for all static fields, they can be used to obtain the equations for any static field you come across. To show how this can be done for different kinds of graviational fields, read on...

3 Using the Concept Relationships to Obtain the Equations for Radial Gravitational Fields

3.1 The Force

Given the relationship between the concepts shown in Figure 3, we can obtain all the equations for these quanitites, so long as we know one of them.

To show you what I mean, I'm going to obtain all the equations for the four quantities, starting from Newton's Law of Gravitation.

Isaac Newton (1643-1727) discovered that in the universe, every object that has mass is attracted by a force to every other object that has mass.

The formula for the force is given by:

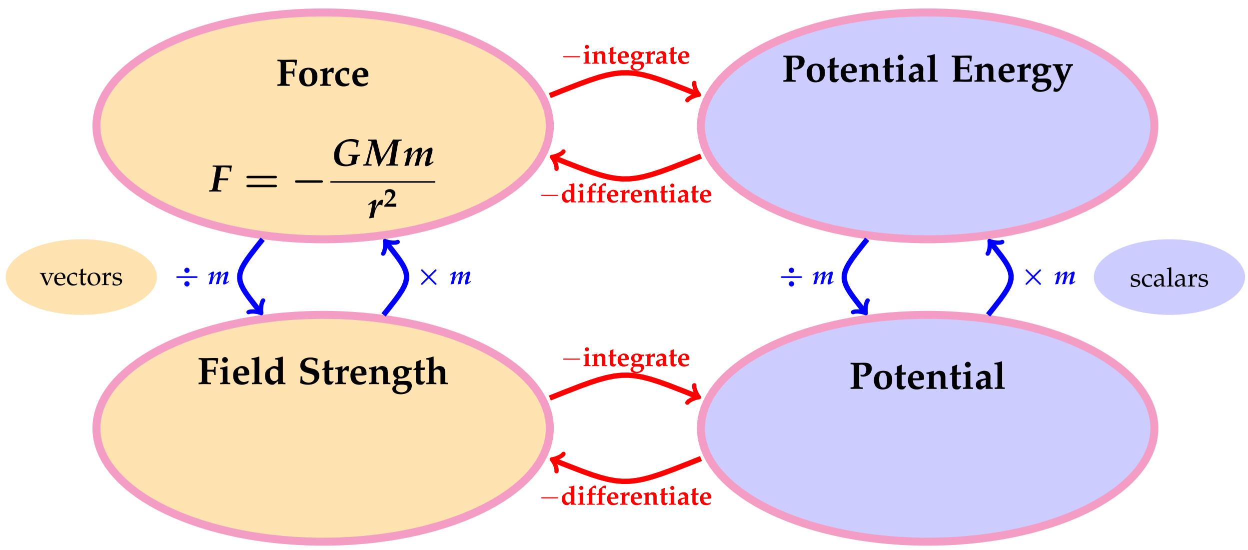
$$F = -\frac{GMm}{r^2} \tag{1}$$

where M and m are the masses of two objects (units: kg) a distance r (m) apart and F is the force (in *Newtons*) between them. G is a constant to make the numbers and units right: it has a value of about 6.67×10^{-11} ($N\ m^2\ kg^{-2}$).

Remember that the $-$ sign crops up because the force is in the opposite direction to the way that the distance is measured (see (Smith, 2014a)).

So, we can include this equation in our picture. See Figure 4.

Figure 4: Obtaining Radial Gravity Equations I: The Force



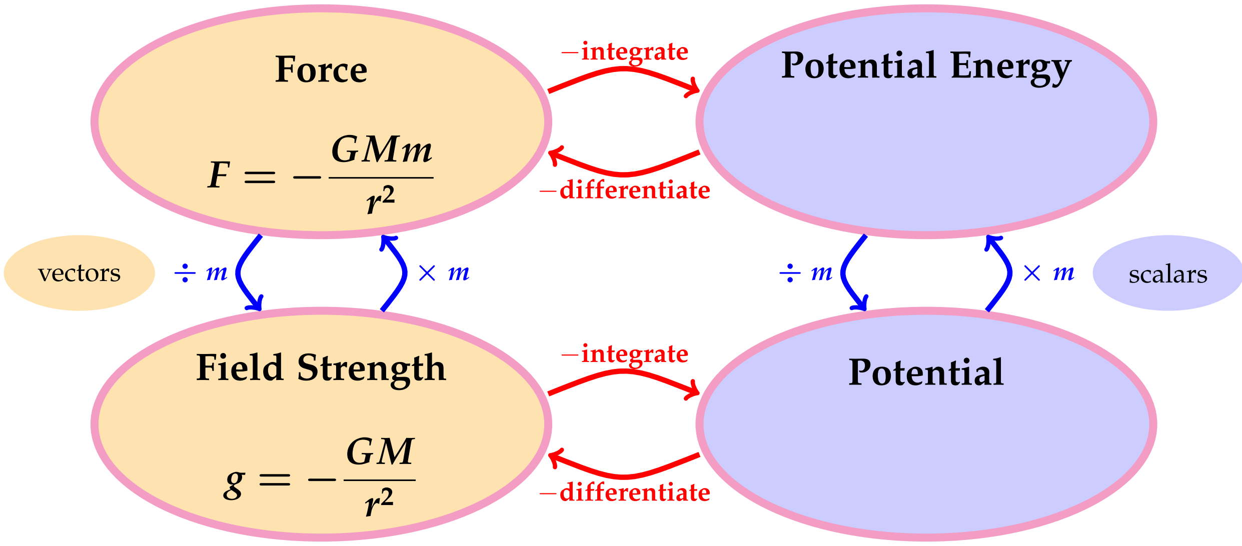
3.2 The Field Strength

So, using Figure 3, and equation 1, we can easily find the field strength. To do that, the picture tells us to take the force and divide it by m:

$$\begin{aligned} g &= -\frac{F}{m} \\ &= -\frac{GMm}{r^2 m} \\ &= -\frac{GM}{r^2} \end{aligned}$$

So we can include the field strength equation into our scheme. See Figure 5.

Figure 5: Obtaining Radial Gravity Equations II: The Field Strength



3.3 The Potential Energy

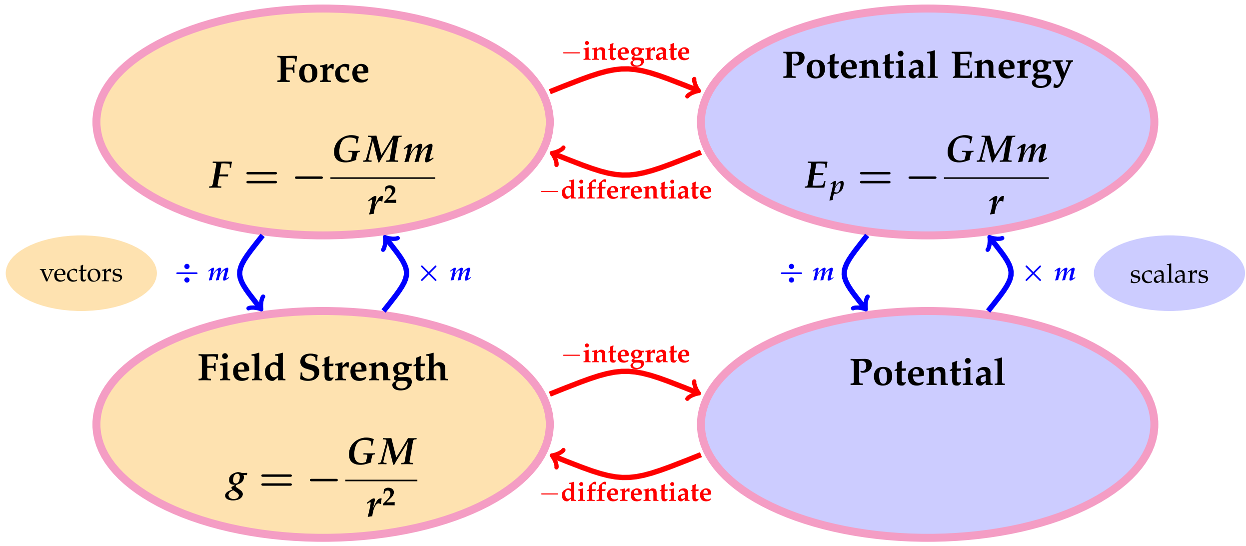
And in a similar way, we can use Figure 3, and equation 1, to find the potential energy equation. To do that, the picture tells us to find the integral of the force, and take the negative of it:

$$\begin{aligned} E_p &= -\int F\,dr \\ &= -\int -\frac{GMm}{r^2}\,dr \\ &= -\frac{GMm}{r} \end{aligned} \tag{2}$$

not forgetting that for radial gravitational fields, we set the constant of integration to zero by convention.

So we can include the potential energy equation into our scheme. See Figure 6.

Figure 6: Obtaining Radial Gravity Equations III: The Potential Energy



3.4 The Potential

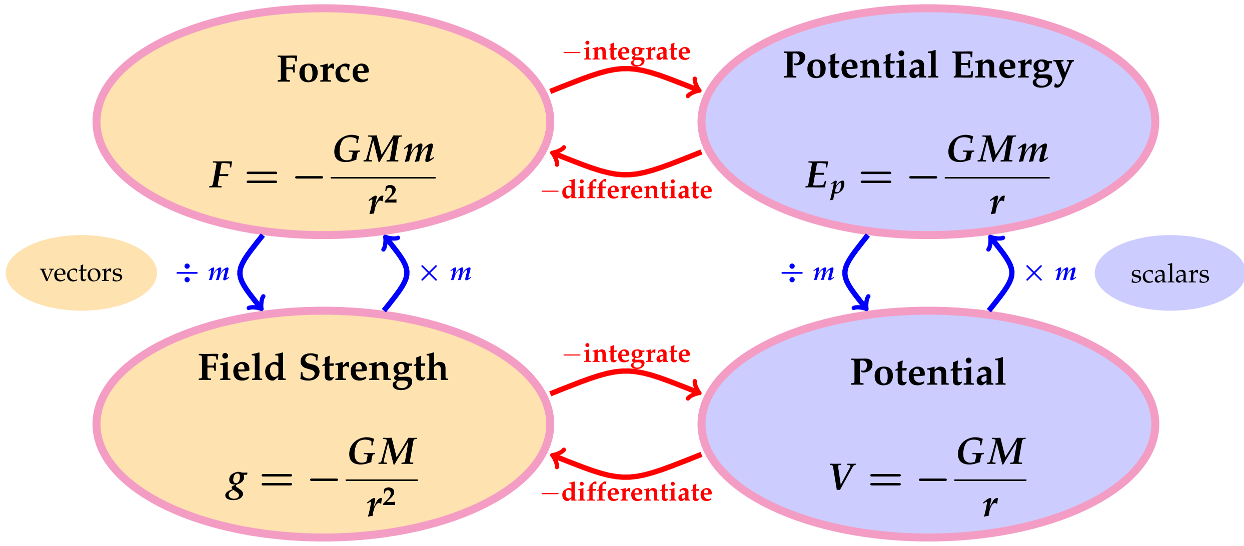
And finally, using Figure 3, and equation 2, we can easily find the potential. To do that, the picture tells us to take the potential energy and divide it by m:

$$
\begin{aligned}
V &= \frac{E_p}{m} \\
&= -\frac{GMm}{rm} \\
&= -\frac{GM}{r}
\end{aligned}
$$

and as with the potential energy, we again set the constant of integration to zero by convention.

So we can include the potential equation into our scheme. See Figure 7.

Figure 7: Obtaining Radial Gravity Equations IV: The Potential



4 Using the Concept Relationships to Obtain the Equations for Uniform Gravitational Fields

"Now, hang on a minute!", I can hear you cry. You can't get uniform gravitational fields, can you? Well no, that's true. But just hear me out...

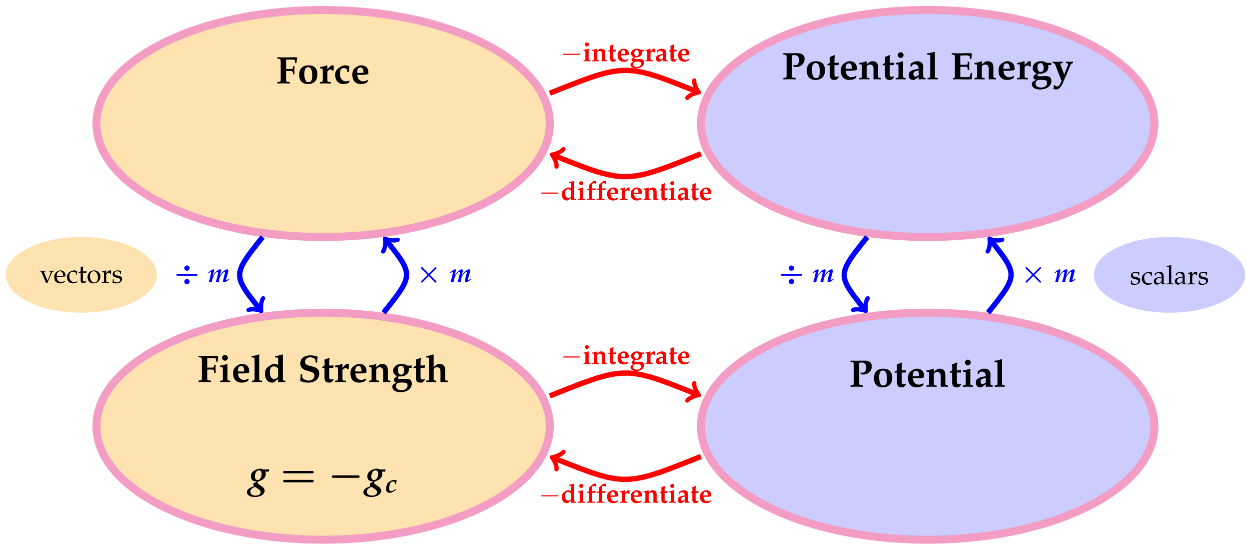
4.1 The Field Strength

Just imagine that there is such a thing as a uniform gravitational field. OK. Well, what about it would be the *uniform* bit?

It turns out that a *uniform* field means that there will be a constant field strength everywhere in the field. So the size of the field strength is the same everywhere, and also the direction of the field strength (and hence the direction of the force) would be the same everywhere.

Alright, so how does this enable us to find the field equations? Well, if we know that the field strength is the same everywhere, then we can fill in the field strength part of the overall picture from Figure 3. See Figure 8.

Figure 8: Obtaining Uniform Gravity Equations I: The Field Strength

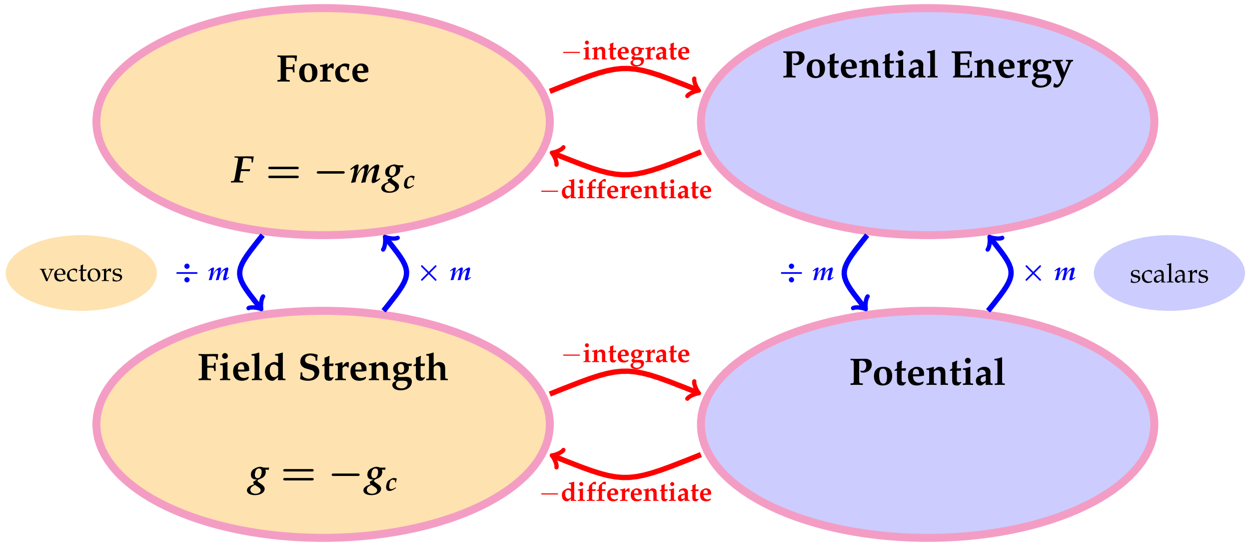


Here. I've denoted by g_c the value of the size of the uniform (i.e. constant) field strength. And did you remember that gravitational fields can only be attractive (see (Smith, 2014a))? Well, that accounts for the − sign.

4.2 The Force

OK, now to get the force, we multiply the field strength by m. So we can fill in the force equation:

Figure 9: Obtaining Uniform Gravity Equations II: The Force



4.3 The Potential Energy

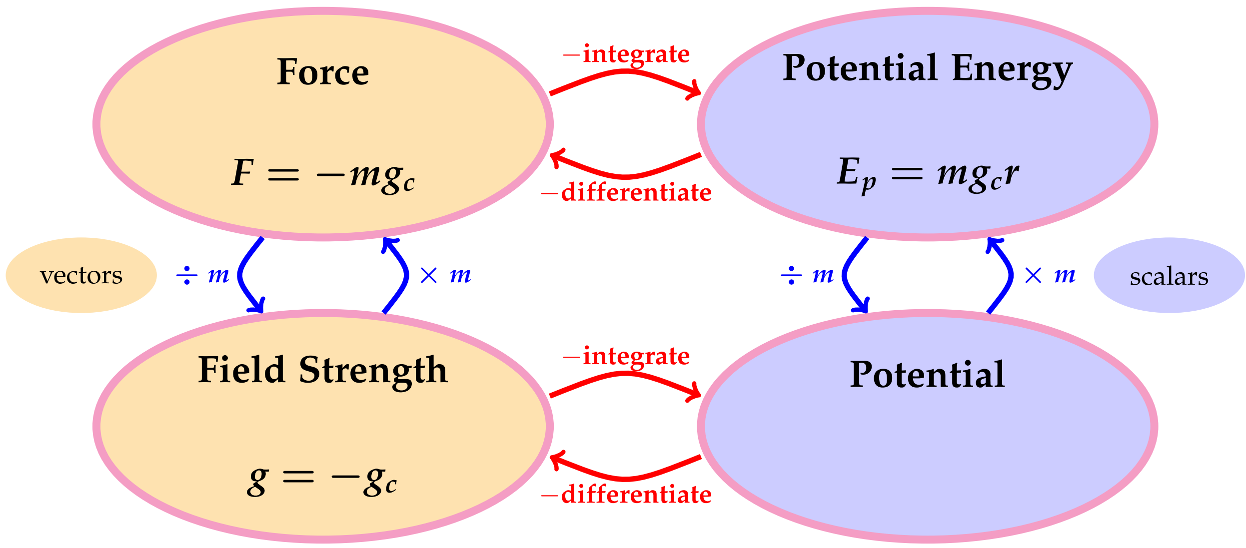
And to find the potential energy formula, we just −integrate the force formula:

$$\begin{aligned} E_p &= -\int F\,dr \\ &= -\int -mg_c\,dr \\ &= mg_c r + E_c \end{aligned} \tag{3}$$

Now again we've got this hassle about the constant of integration. What's the convention we use this time? Well, in the case of a uniform gravitational field, we usually take the zero of potential energy to be the place where r is zero (which is very often the *ground*). That would mean that the constant of integration would again be zero. Phew!

So we can include the potential energy equation into our scheme. See Figure 10.

Figure 10: Obtaining Uniform Gravity Equations III: The Potential Energy



4.4 The Potential

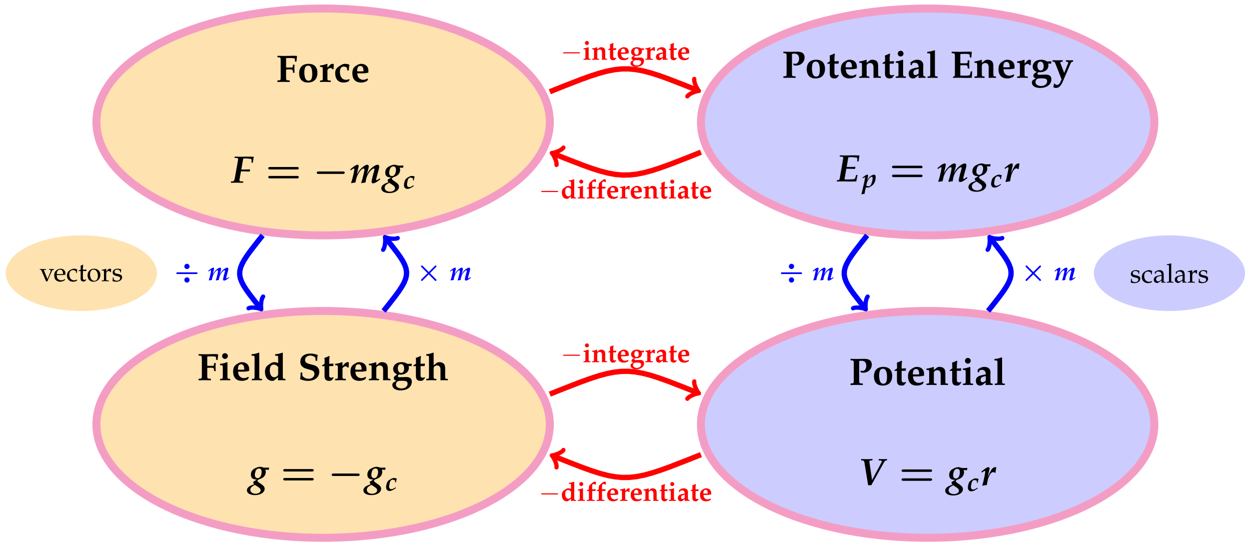
And finally, we can easily find the potential. To do that, the picture tells us to take the potential energy and divide it by m:

$$\begin{aligned} V &= \frac{E_p}{m} \\ &= \frac{mg_c r}{m} \\ &= g_c r \end{aligned}$$

and as with the case of the potential energy, the constant of integration will again be zero if we choose our zero value of potential energy to be the ground (where $r = 0$).

So we can include the potential equation into our scheme. See Figure 11.

Figure 11: Obtaining Uniform Gravity Equations IV: The Potential



4.5 Pause for Thought...

Just have a look at the equations in Figure 11. Does $F = mg_c$ remind you of anything? $F = mg$ perhaps?

And what about $E_p = mg_c r$? Does that remind you of $E_p = mgh$ perhaps?

In fact, these are the equations that we've used right up until recently to solve problems with a gravitational field involved. Gravitational potential energy questions. Projectiles questions. SUVAT questions.

All of these kinds of questions had implicitly assumed *a uniform gravitational field*, without explicitly saying so. And we never batted an eyelid...

5 Appendices

A Useful Data

Quantity	Value
Earth - Moon Distance	3.844×10^{8} m
Gravitational Constant	6.673×10^{-11} m^{3} kg^{-1} s^{-2}
Mass of the Earth	5.972×10^{24} kg
Radius of the Earth	6.371×10^{6} m
Mass of the Moon	7.347×10^{22} kg
Gravitational Field Strength at the surface of the Earth	9.81 ms^{-2}
k	8.9876×10^{9} m^{3} kg s^{-4} A^{-2}
ϵ	8.8542×10^{-12} m^{-3} kg^{-1} s^{4} A^{2}

References

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Smith, S. (2014e). Static Fields V: Electric Field Examples. Questions to practice electric field calculations.

Smith, S. (2015). Dynamic Fields: Magnetic Fields. Explains how magnetic fields work.