Steve Smith Tuition: Physics Notes

Static Fields III: Electric Field Essentials

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Prerequisites

I'm assuming here that you have had the stamina to read through my document Smith (2014a). This explains the relationships between all the key field concepts, and where the equations come from.

If you haven't read that document, you probably should, but if you want to cut to the chase, and you want the short route to answering fields questions, then so long as you are prepared to take a few things on trust, this is the document for you!

It would also be nice if you had read my document Smith (2014b). This explains how to obtain all the radial and uniform gravitational field equations if you know the relationship between the key concepts.

Notes

None.

Document History

Date	Version	Comments
18th October 2014	1.0	Initial creation of the document.
28th March 2015	1.1	Simplifying uniform electric fields.

1 Introduction

Here, we are only interested in static fields. That is, fields that don't change with time. There are three static fields that A-Level students encounter: gravitational fields, electric fields and magnetic fields. This document is only concerned with electric fields. For a discussion of gravitational fields see Ref (Smith, 2014a), and for magnetic fields, see Ref (Smith, 2015).

1.1 What is a Field?

In physics, a field is simply a region of space where an object of a particular kind would experience a force. What I mean by *a particular kind* is that objects experience forces because of some property that they have. For example, an object that has *mass* experiences a *gravitational* force when located in a *gravitational* field. And an object that has *charge* experiences an *electric* force when located in an *electric* field. Interestingly, gravitational fields are caused by masses, and electric fields are caused by electric charges.

1.2 The Two Key Questions

When analysing fields, there are only two questions that we need to answer. These are:

- **What is the force on my object when it is placed at a particular point in the field?**
- **How much energy will it take to move my object from one point to another in the field?**

When you study anything to do with fields, these are the two things to have uppermost in your mind as you read. Keep asking yourself these two questions. Everything about fields boils down to these two key questions. As you read on, don't forget to keep these questions in mind. Have I emphasised that enough? You know, I'm not sure I have! What were those questions again...?

The Two Key Questions:

(1) What is the force on my object at a particular point in the field?

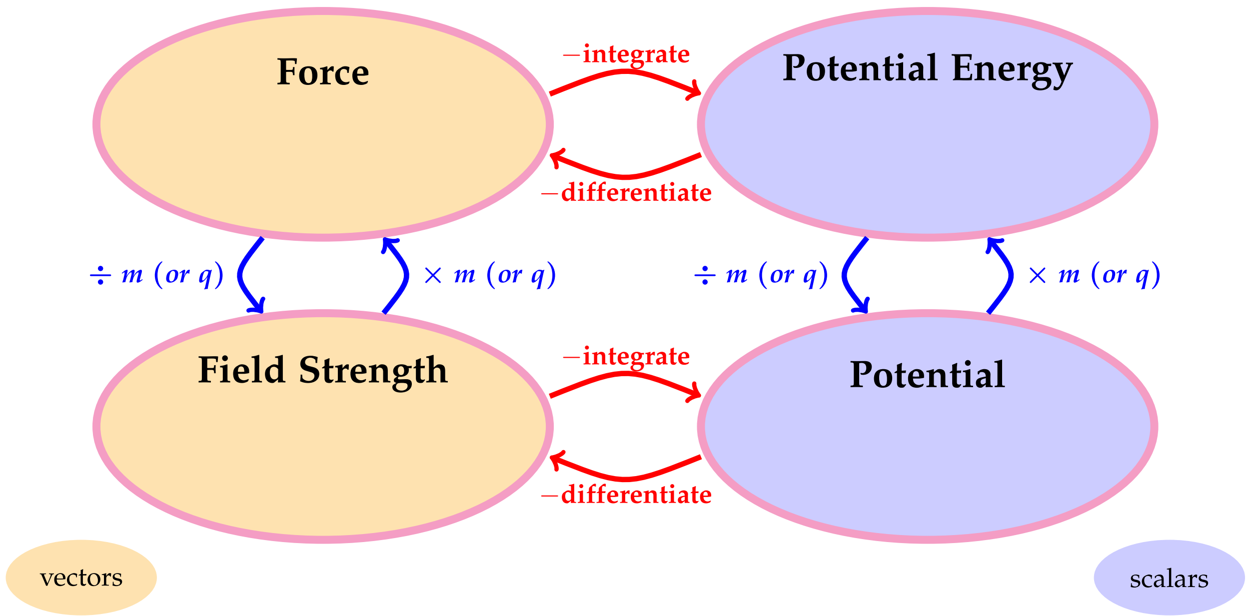
(2) How much energy would it take to move my object from one point to another in the field?

So to start answering these questions, let's look at one of the fields we are interested in at A-Level. Let's start by recapping on a subject with some gravity...

2 The Relationship Between the Key Concepts

In (Smith, 2014a), I presented the picture representing the four key concepts for gravitational fields. Here is the same diagram, but this time I've removed the equations so the concepts themselves stand out.

Figure 1: Summary of Fields



The reason for the m $(or\ q)$ business is that the thing you have to multiply or divide by depends on the type of field you have. Gravitational fields affect masses, so you would use m if you were looking at a gravitational field; electric fields affect charges, so you would use q if you were looking at an electric field.

Because these relationships are the same for all static fields, they can be used to obtain the equations for any static field you come across. To show how this can be done for different kinds of electric fields, read on...

3 Using the Concept Relationships to Obtain the Equations for Radial Electric Fields

3.1 The Force

Given the relationship between the concepts shown in Figure 1, we can obtain all the equations for these quanitites, so long as we know one of them.

To show you what I mean, I'm going to obtain all the equations for the four quantities, starting from Coulomb's Force Law.

Charles Augustin de Coulomb (1736-1806) discovered in 1785 that in the universe, there is a force between any two objects that have charge.

The formula for the size of the force is given by:

$$F = \frac{Qq}{4\pi\epsilon r^2} \tag{1}$$

where Q and q are the charges on two objects (units: C) a distance r (m) apart and F is the force (in *Newtons*) between them. ϵ is a constant to make the numbers and units right: it has a value of about 8.85×10^{-12} (m^{-3} kg^{-1} s^4 A^2).

These days, a slightly simplified version of the formula is used, grouping the $\frac{1}{4\pi\epsilon}$ together in just one constant k:

$$F = \frac{kQq}{r^2} \tag{2}$$

the value of k being about 8.99×10^9 (N m^2 C^{-2}). Throughout the rest of this document, I'll be using Equation (2). If you follow a course that uses Equation (1), I apologise, but I'm sure you will adapt. Just put $\frac{1}{4\pi\epsilon}$ wherever there is a k.

3.1.1 Please, Give Me a Sign...

Now I have to pause here for a moment to discuss the *sign* in front of this equation. What sign, you may ask? Exactly, I would reply!

If you have read (Smith, 2014a), you should be aware of a difficulty that arises when studying fields. The difficulty I'm talking about is the one that arises because force and field strength are *vector* quantities, and so *direction* of these things is important.

A way of handling that is the convention that attractive forces are negative, and repulsive forces are positive.

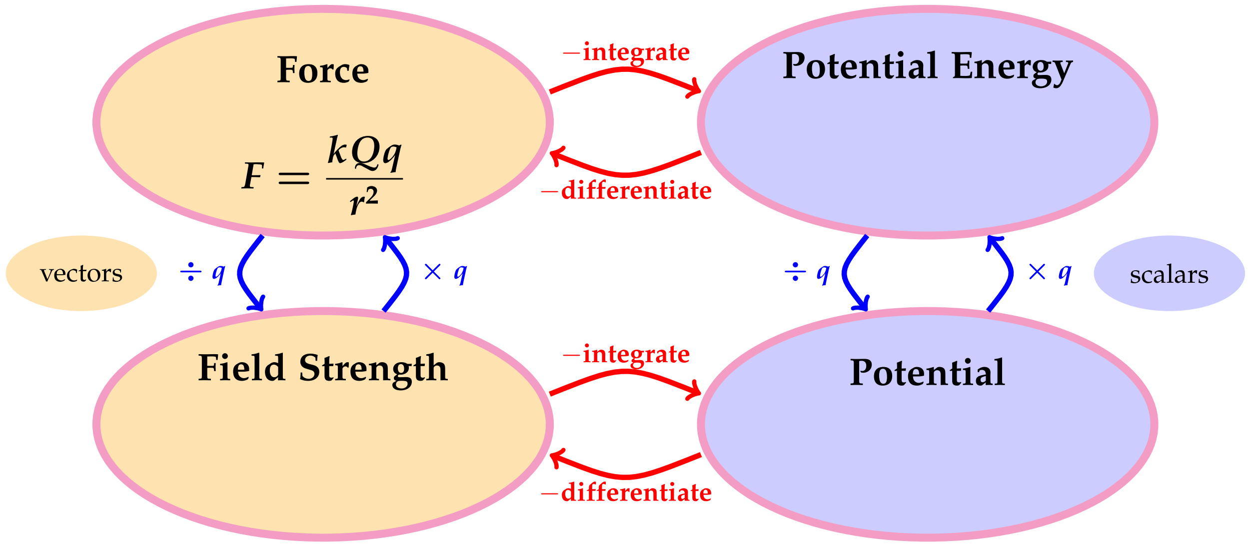
Now, how does that work here? Well, let's say that we had two charged particles, one with a positive charge, and the other with a negative charge, such as a proton and an electron. Now opposite charges attract, right? So the force between them will be attractive. So by our convention, we need the force to be negative. But in Coulomb's force equation, the constant and the r are positive things. So if one of the charges, Q say, was negative, and the other, q, was positive, then the whole expression will be negative! Exactly what we want!

And let's say that we had two charged particles, both with a positive charge, such as two protons. Now similar charges repel, right? So the force between them will be repulsive. So by our convention, we need the force to be positive. But in Coulomb's force equation, the constant and the r are positive things. So if both of the charges, Q and q are positive, then the whole expression will be positive! Exactly what we want!

So we don't need a negative sign in front of the expression, like we did with the gravitational force formula.

After all that, we can include this equation in our picture. See Figure 2.

Figure 2: Obtaining Radial Electric Field Equations I: The Force



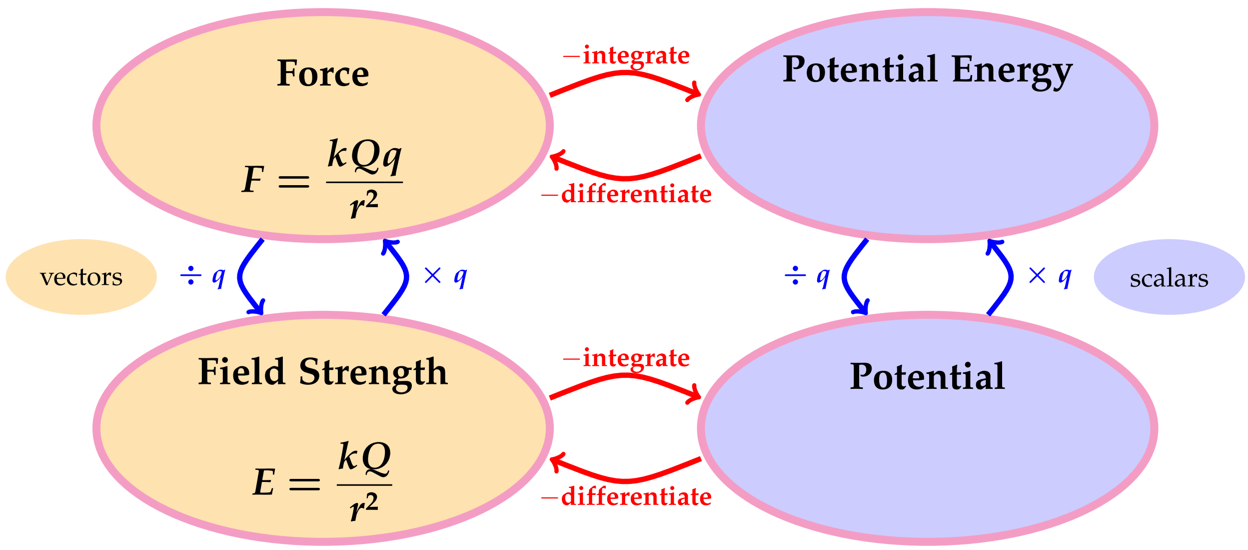
3.2 The Field Strength

So, using Figure 1, and equation (2), we can easily find the field strength. To do that, the picture tells us to take the force and divide it by q:

$$\begin{aligned} E &= \frac{F}{q} \\ &= \frac{kQq}{r^2 q} \\ &= \frac{kQ}{r^2} \end{aligned}$$

So we can include the field strength equation into our scheme. See Figure 3.

Figure 3: Obtaining Radial Electric Field Equations II: The Field Strength



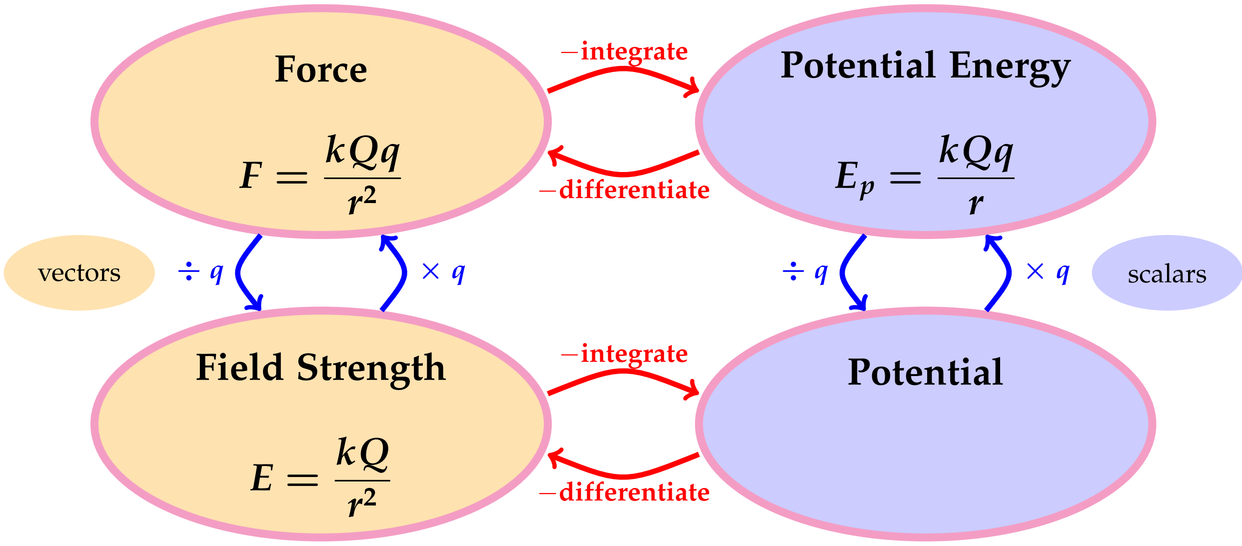
3.3 The Potential Energy

And in a similar way, we can use Figure 1, and equation (2), to find the potential energy equation. To do that, the picture tells us to find the integral of the force, and take the negative of it[1]:

$$\begin{aligned} E_p &= -\int F\,dr \\ &= -\int \frac{kQq}{r^2}\,dr \\ &= \frac{kQq}{r} \end{aligned} \tag{3}$$

So we can include the potential energy equation into our scheme. See Figure 4.

Figure 4: Obtaining Radial Electric Field Equations III: The Potential Energy



Notice here that in the case of a *radial* electric field, the constant of the integration process is taken to be zero by convention, just as it was in the case of a radial gravitational field.

[1] By convention, the constant of integration is taken to be zero here. See (Smith, 2014a) for a full explation of this.

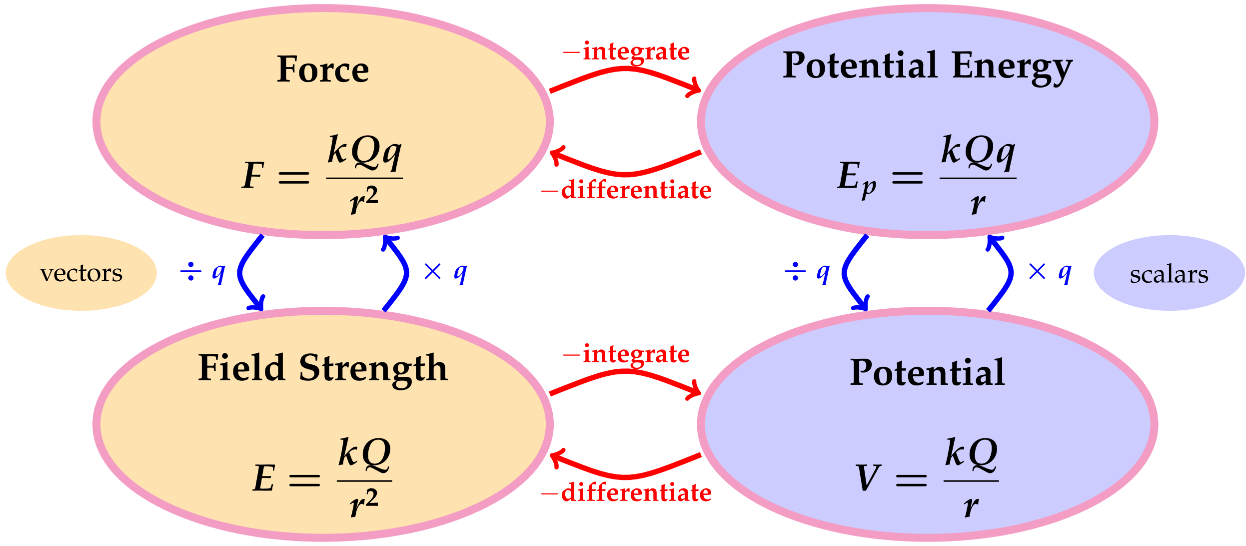
3.4 The Potential

And finally, using Figure 1, and equation 3, we can easily find the potential. To do that, the picture tells us to take the potential energy and divide it by q:

$$\begin{aligned} V &= \frac{E_p}{q} \\ &= \frac{kQq}{rq} \\ &= \frac{kQ}{r} \end{aligned}$$

So we can include the potential equation into our scheme. See Figure 5.

Figure 5: Obtaining Radial Electric Field Equations IV: The Full Monty



Again, the constant of integration in the potential equation is taken to be zero by convention of a radial electric field.

4 Using the Concept Relationships to Obtain the Equations for Uniform Electric Fields

4.1 The Nature of Uniform Fields...1

Before we have a look at where the uniform electric field equations come from, let's have a look at how uniform electric fields are created, and the different possibilities we can get.

Uniform electric fields are formed in the space between conducting plates which are held at different potentials. See for example, Figure 6. In this picture, the black lines represent the plates. In this case, one

Figure 6: A Uniform Electric Field

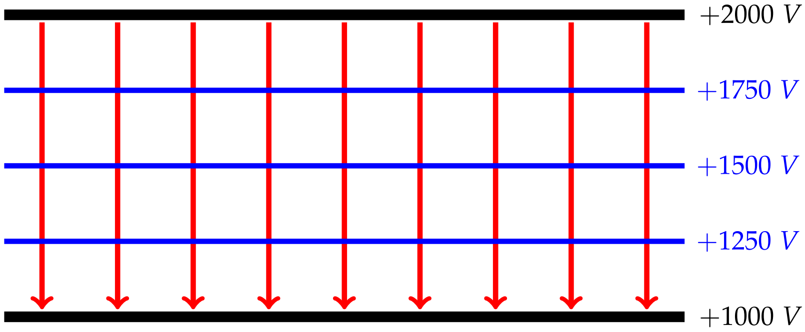


plate is held at +1000 V, while the other plate is held at a potential of +2000 V. The red lines indicate the electric field lines (going down because they show the direction a small *positive* charge would move due to the field), and the blue lines show some equipotential lines in this field.

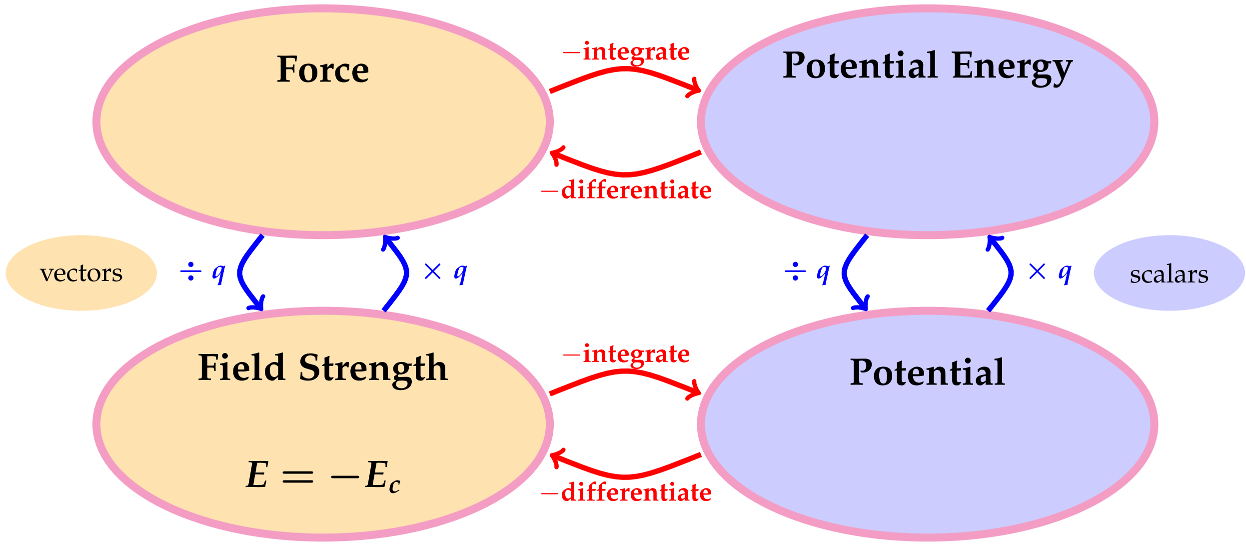
Now this is very analogous to the uniform gravitational field that we experienced in Smith (2014b). If we measure distance in this field (r) going *up* from the lower plate, then the electric field direction is opposite to the way that we measure distance, and so a negative sign would be introduced into the field strength equation...

4.2 The Field Strength

The definition of a uniform field is one where the field strength is constant everywhere. That means the size of the field strength is the same everywhere, and the direction of the field strength is the same everywhere. Alright, so how does this enable us to find the field equations? Well, if we know that the field strength is the same everywhere, then we can fill in the field strength part of the overall picture from Figure 1. See Figure 7.

Here. I've denoted by $-E_c$ the value of the size of the uniform (i.e. constant) field strength (with a negative sign because we are measuring the distance r up).

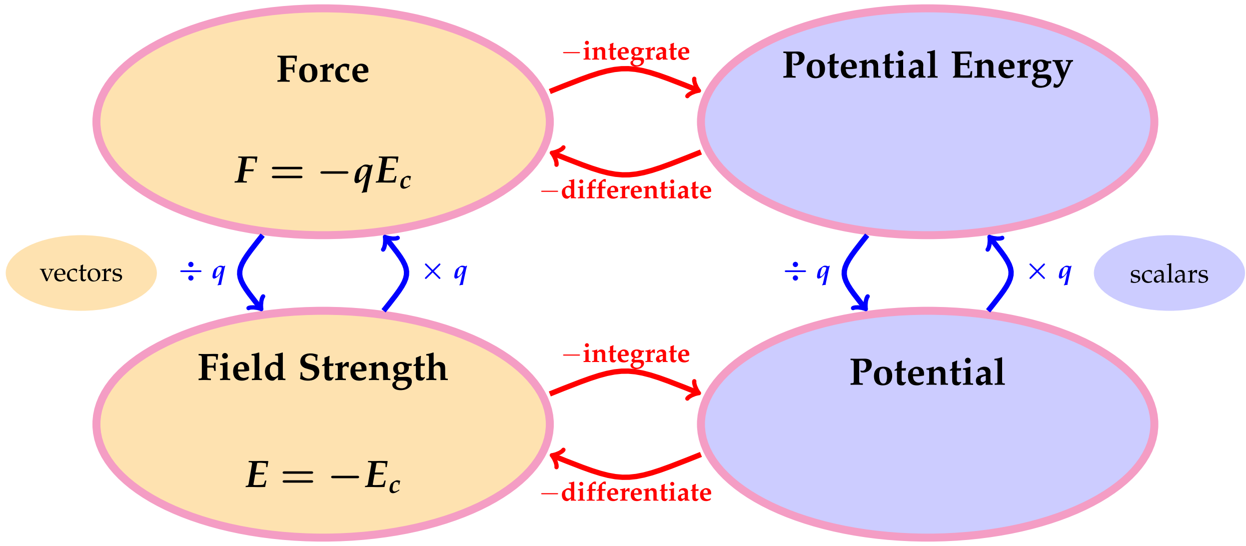
Figure 7: Obtaining Uniform Electric Field Equations I: The Field Strength



4.3 The Force

OK, now to get the force, we multiply the field strength by q. So we can fill in the force equation:

Figure 8: Obtaining Uniform Electric Field Equations II: The Force



4.4 The Potential

Now at this point the next thing I'm going to do is to find the *potential*. To do that, the picture tells us to take the electric field strength, and −integrate it:

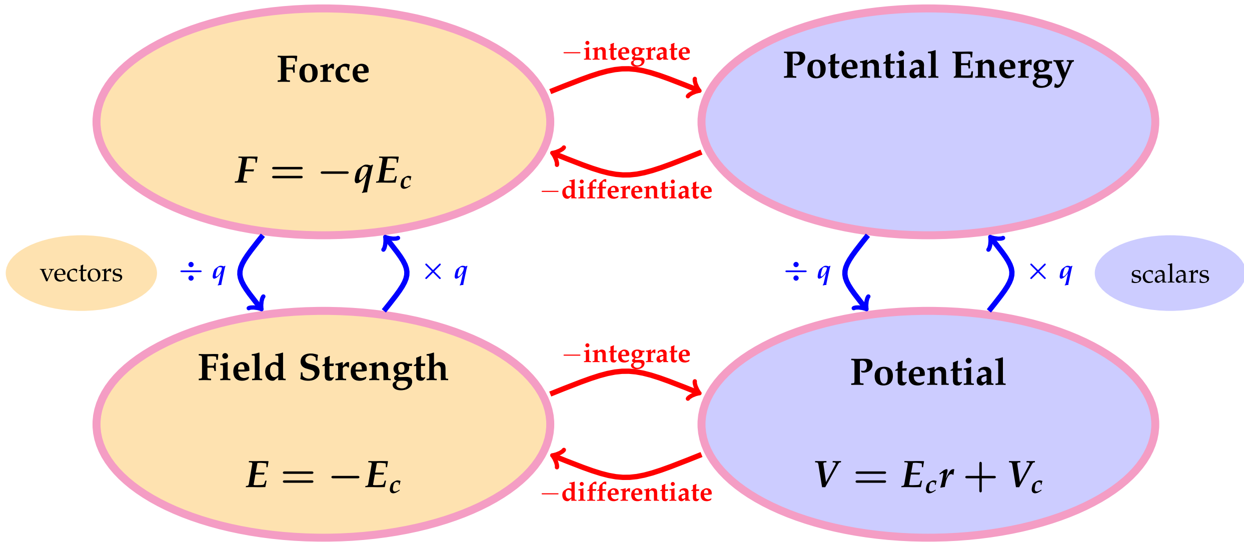
$$\begin{aligned} V &= -\int E_c\, dr \\ &= E_c r + V_c \end{aligned} \tag{4}$$

Now where did this V_c come from? It's the *constant of integration*. And how do we find the constant of integration? You have to know what the potential is for some value of r. Looking at Figure 6, we know that the potential is $+1000\ V$ when $r = 0$. So

$$\begin{aligned} 1000 &= E_c \times 0 + V_c \\ \Rightarrow \quad V_c &= 1000 \end{aligned}$$

So we can include the potential equation into our scheme. See Figure 9.

Figure 9: Obtaining Uniform Electric Field Equations III: The Potential



Most of the time, we are only interested in a *potential difference* when doing calculations concerning uniform electric fields. In that case we can ignore the constant of integration, just as we do when we have limits on an integral: the constant of integration cancels out. However, if you wanted the absolute potential for a particular value of r, then Equation (4) will be the one to use.

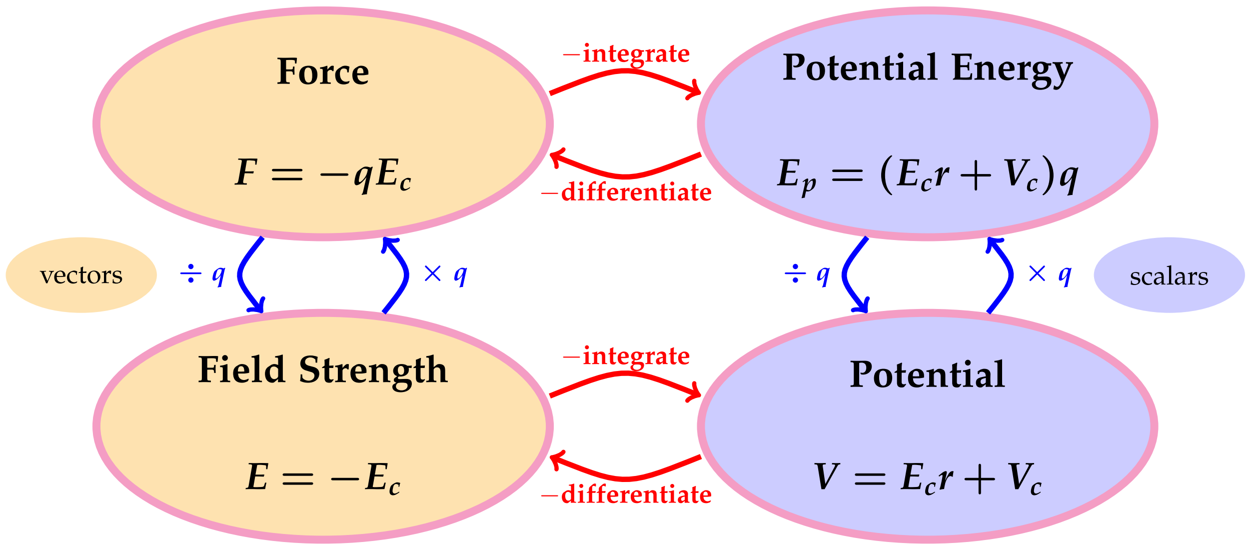
4.5 The Potential Energy

And to find the potential energy formula, we just multiply the potential formula by q:

$$E_p = (E_c r + V_c)q \tag{5}$$

So we can include the potential energy equation into our scheme. See Figure 10.

Figure 10: Obtaining Uniform Electric Field Equations IV: The Full Monty

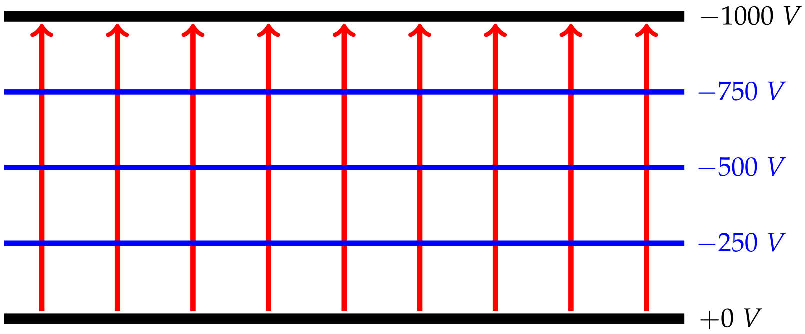


Again, most of the time, we are only interested in a *potential energy difference* when doing calculations concerning uniform electric fields. In that case we can ignore the constant of integration, just as we do when we have limits on an integral: the constant of integration cancels out. However, if you wanted the absolute potential energy for a particular value of r, then Equation (5) will be the one to use.

4.6 The Nature of Uniform Fields...2

...but then we can get uniform electric fields such as that shown in Figure 11. In this picture, the black

Figure 11: Another Uniform Electric Field



lines represent the plates. In this case, one plate is held at 0 V ("Earthed" it is called), while the other plate is held at a potential of -1000 V. The red lines indicate the electric field lines (going up because they show the direction a small *positive* charge would move due to the field), and the blue lines show some equipotential lines in this field.

To keep things as simple as we can, just treat this field in the same way that we did in Section 4.1: think of things relative to the *top* plate this time (the one where the field lines point). Then if we measure distance *down* from there, then the electric field direction is again in the *opposite* direction as the way that we measure distance, and so we again have a negative sign in the field strength equation, and all the equations will be the same as before.

5 Appendices

A Useful Data

Quantity	Value
Earth - Moon Distance	3.844×10^{8} m
Gravitational Constant	6.673×10^{-11} m^{3} kg^{-1} s^{-2}
Mass of the Earth	5.972×10^{24} kg
Radius of the Earth	6.371×10^{6} m
Mass of the Moon	7.347×10^{22} kg
Gravitational Field Strength at the surface of the Earth	9.81 ms^{-2}
k	8.9876×10^{9} m^{3} kg s^{-4} A^{-2}
ϵ	8.8542×10^{-12} m^{-3} kg^{-1} s^{4} A^{2}

References

Smith, S. (2014a). Static Fields I: Gravity. An introduction to fields through Newton's Law of Gravity.

Smith, S. (2014b). Static Fields II: Gravity Essentials. How to obtain all the gravitational field equations.

Smith, S. (2015). Dynamic Fields: Magnetic Fields. Explains how magnetic fields work.