Steve Smith Tuition: Physics Notes

Phase, Phase Difference and Path Difference

Contents

Prerequisites

Before reading this, you should really have some idea about the basics of waves: all the big ideas and the terms involved, like *wavelength*, *amplitude*, *frequency*, *cycle*, *period*, etc., etc. If you don't know about any of this stuff, go and have a look at (Smith, 2015a).

You should also know about *superposition of waves*. If you don't, check out (Smith, 2015b).

Notes

None.

Document History

Date	Version	Comments
16th April 2015	1.0	Initial creation of the document.

1 Phase and Phase Difference

Here is a picture of a wave.

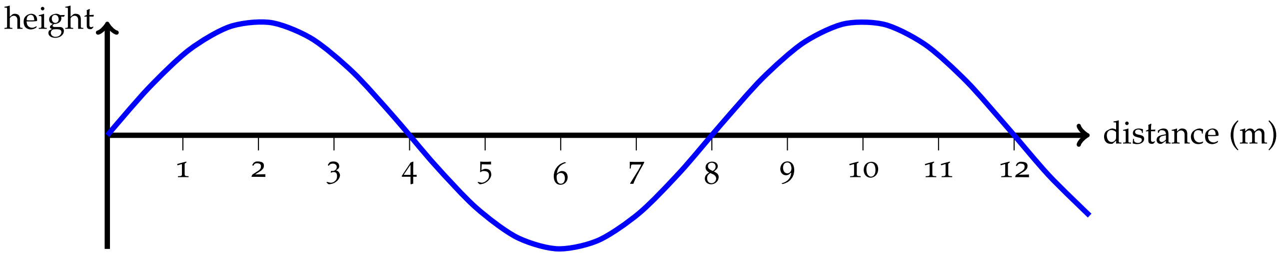

Figure 1: A Wave

And here is a picture of another wave, drawn on the same diagram as the first wave.

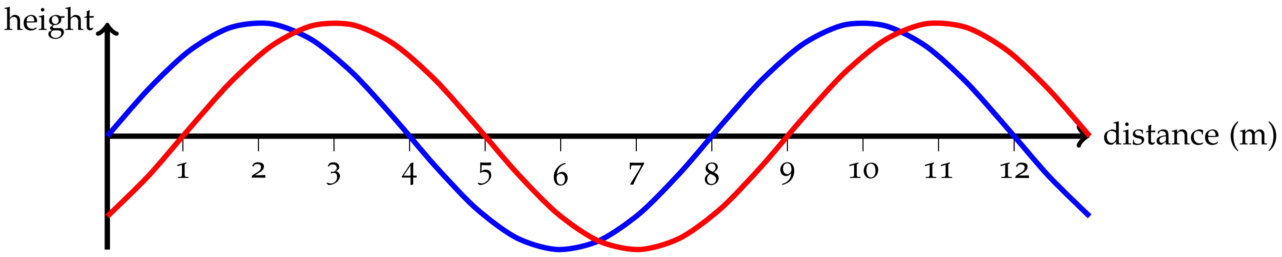

Figure 2: Two Waves

So - what's the difference between these two waves?

I've tried to draw them so that both waves have the same amplitude and wavelength. And I'll tell you that I want these two waves to have the same frequency (and hence period) and velocity as well. So everything's the same about these two waves, right?

Well, not quite, obviously. The only difference between these two waves is that one is *shifted* a bit along the distance axis relative to the other one.

1.1 How Big is the Shift?

So the next question to ask would be "How big is the shift between the waves?". And this question leads on to a fair amount of confusion, because there is a number of different ways of answering this question. Which is a pity, because the idea of shifting one wave relative to another is quite a simple one.

1.1.1 Using Distance to Measure the Shift

In order to determine out what the shift will be, we have to look at corresponding points on the two waves. On Figure 3 I have marked three pairs of points that represent equivalent points on the two waves. The suffix b just signifies that the point is on the blue wave; The suffix r just signifies that the point is on the red wave. I've also marked on Figure 3 the shifts between these points.

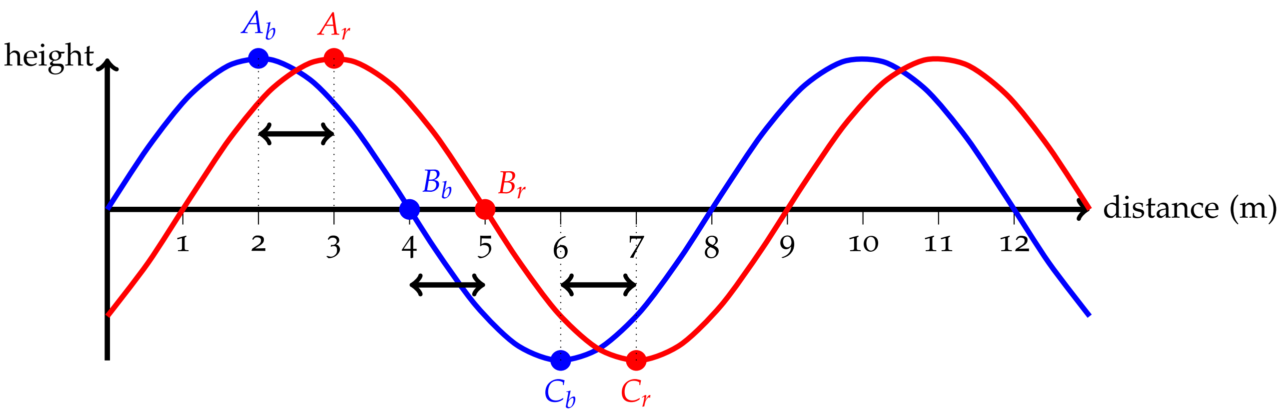

Figure 3: Using Distance as a Measure of the Shift

I hope you can see that the red wave has been shifted 1 m to the right of the blue wave. So one answer to the question "How big is the shift between the waves?" would be: "The shift between the waves is 1 m".

1.1.2 "Phase" and "Phase Difference"

Scientists don't use the term *shift* when they are talking about this sort of thing. Pity, as that's all it is: one wave shifted relative to another.

Instead they use the term *phase difference*. What *phase* is, I'll talk about later (see Appendix A).

So our question should really be "How big is the phase difference between these waves?" And one answer to that question should be: "The phase difference between the waves is 1 m".

1.1.3 Using Time to Measure Phase Difference

Believe it or not, Figure 4 is a picture of *the same two waves*! How can this be? The answer lies in the horizontal axis. Figure 4 is a picture of the same waves but drawn with *time* as the horizontal

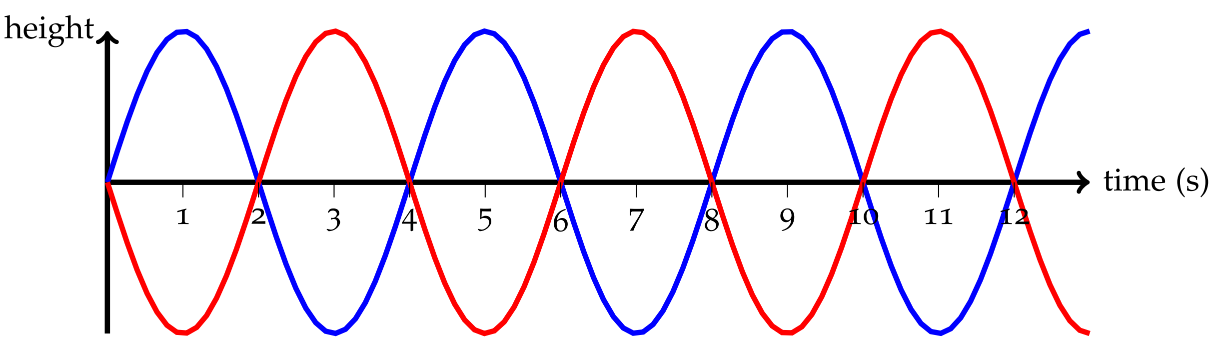

Figure 4: The Same Two Waves!

axis instead of *distance*. And look: *the phase difference is different*!!

This time, the red wave has been shifted 2 units to the right of the blue wave. And because the horizontal axis measures *time*, that corresponds to 2 s.

So, another answer to the question "How big is the phase difference between these waves?" could be: "The phase difference between the waves is 2 s".

If you found this section bewildering, see (Smith, 2015a).

1.1.4 A Short Digression Into the World of Maths...

I want to come away from talking about phase difference for a moment, because I want to recap a bit of Maths that you may have forgotten. And it's a bit of Maths that nobody likes much. It's drawing a graph of the sine function. Urgh!

Figure 5 is a picture of the sine function. When you draw the sine function in Maths, the hori-

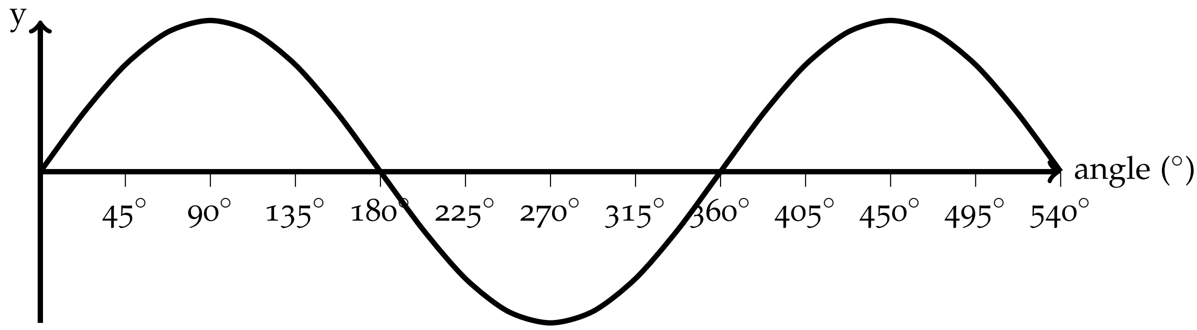

Figure 5: The sine function

zontal axis is *angle*. That's because the sine function takes an angle as it's input. Remember? For example,

$$\sin(30°) = \frac{1}{2}$$

When you type sin(30) into your calculator, the 30 is an *angle*.

1.1.5 Using Angle to Measure the Shift

Now if you have another look at Figure 3, the blue wave looks just like a sine function. So actually, I could re-draw Figure 3, changing the horizontal axis from *distance* to *angle*. Figure 6 is what I mean. So if we have a look at Figure 6 to determine what the phase difference between the two waves is, then we would come up with the answer 45°!

So yet another answer to the question "How big is the phase difference between the waves?" would be: "The phase difference between the waves is 45°".

1.1.6 Using Wavelength to Measure the Shift

And you know what? *There's yet another way to measure phase difference!!*

I'm going to draw Figure 3 again, this time adding the wavelength (λ) of the blue wave. Remember that the wavelengths of these waves are the same, so that would be the wavelength of the red wave too.

In Figure 7 I've made another change: I've changed the horizontal axis yet again: this time to *wavelengths*. Since one whole wavelength of both waves takes up eight "ticks" on the horizontal axis, each "tick" must represent an eighth of a wavelength.

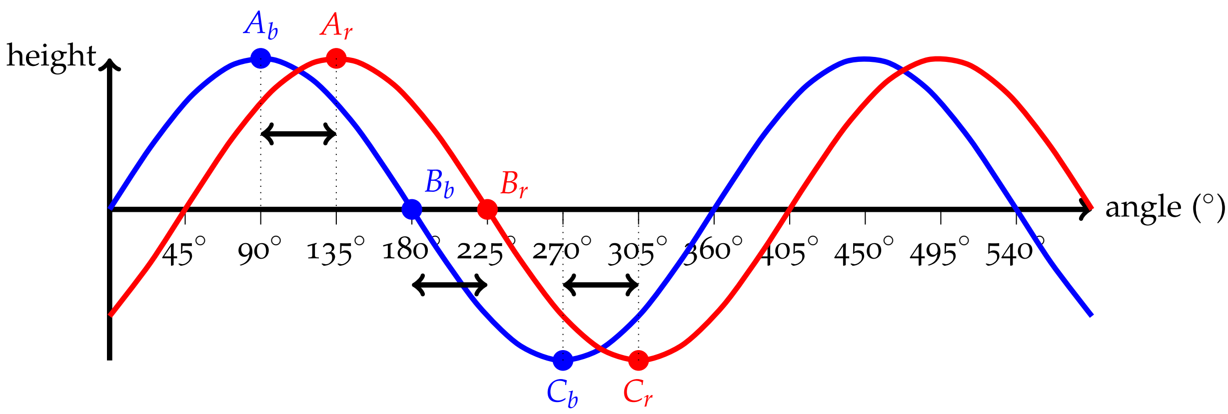

Figure 6: Using Angle as a Measure of the Shift

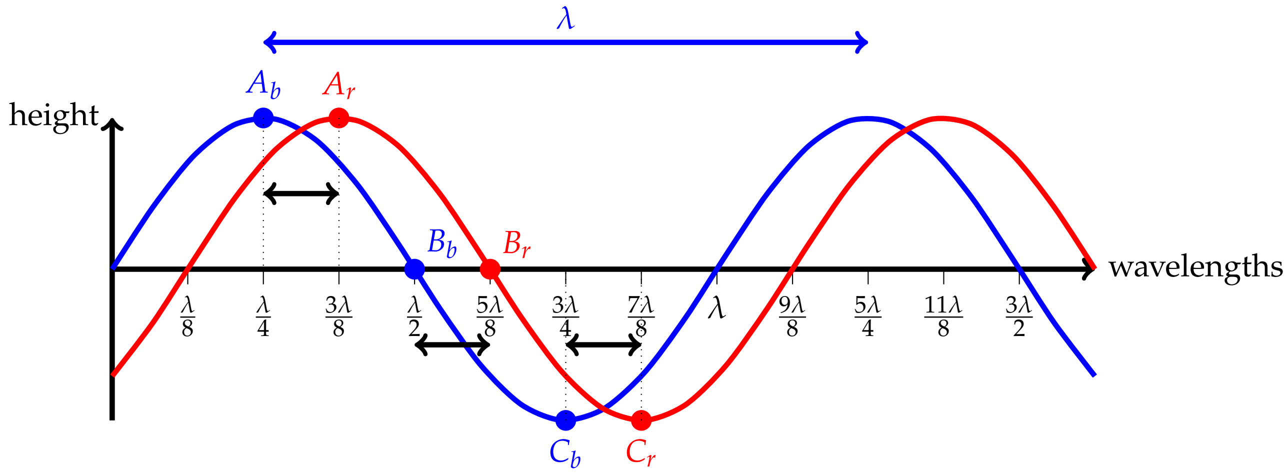

Figure 7: Using Wavelength as a Measure of the Shift

So if we have a look at Figure 7 to determine what the phase difference between the two waves is, then we would come up with the answer $\frac{\lambda}{8}$!

So yet another answer to the question "How big is the phase difference between the waves?" would be: "The phase difference between the waves is $\frac{\lambda}{8}$".

1.2 Summary of Phase Difference

So, to sum up: it's possible to measure the phase difference between two waves using *all* of these quantities:

- distance;
- time;
- angle[1];
- wavelengths.

Which is best? Impossible to say, as it will depend on the situation. But a rule of thumb would be: don't use distance or time; use angle or wavelengths to measure phase difference. And I'd probably prefer angle in most situations.

However, wavelengths is the right option when we want to consider *path difference*...

[1]And you know what? If you do A-Level Maths you will know that you can measure angle in two ways: using degrees or radians!

2 Path Difference

Imagine that we have two waves that start out in phase. That is, they start out with a phase difference of 0 (in whatever units). See, for example, Figure 8. Let's say these waves were light.

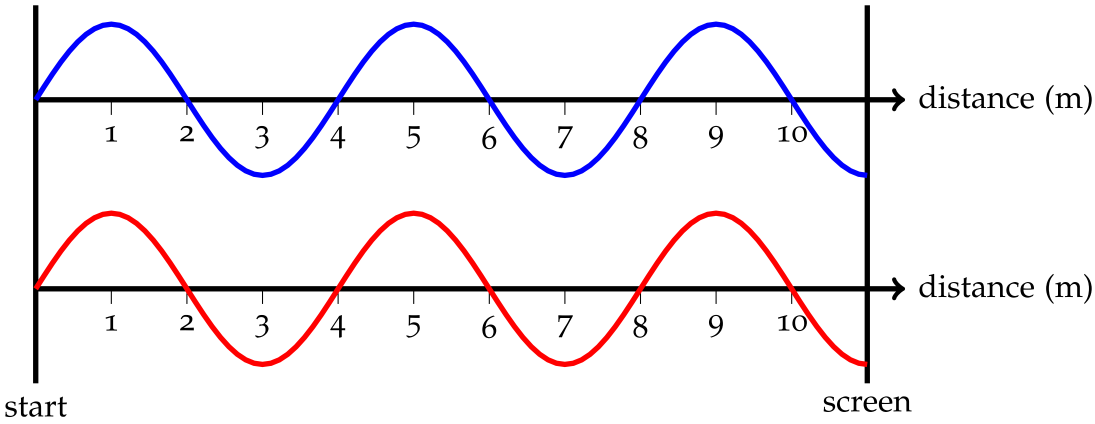

Figure 8: Two Waves Always In Phase

And let's also say that these light waves were pointing to the same place on a screen, so that both waves hit the screen at the same point. We are interested in what we see on the screen.

Well, if, as in the case of Figure 8, *the two waves have the same distance to travel*, then they will always be in phase, and they will hit the screen in phase, and so there will be a bright spot on the screen, since the two waves will constructively interfere there (see Smith (2015b)).

However, look at what happens if the two waves have different distances to travel (See Figure 9). The red wave has two more meters to travel than the blue wave, so even though the two waves

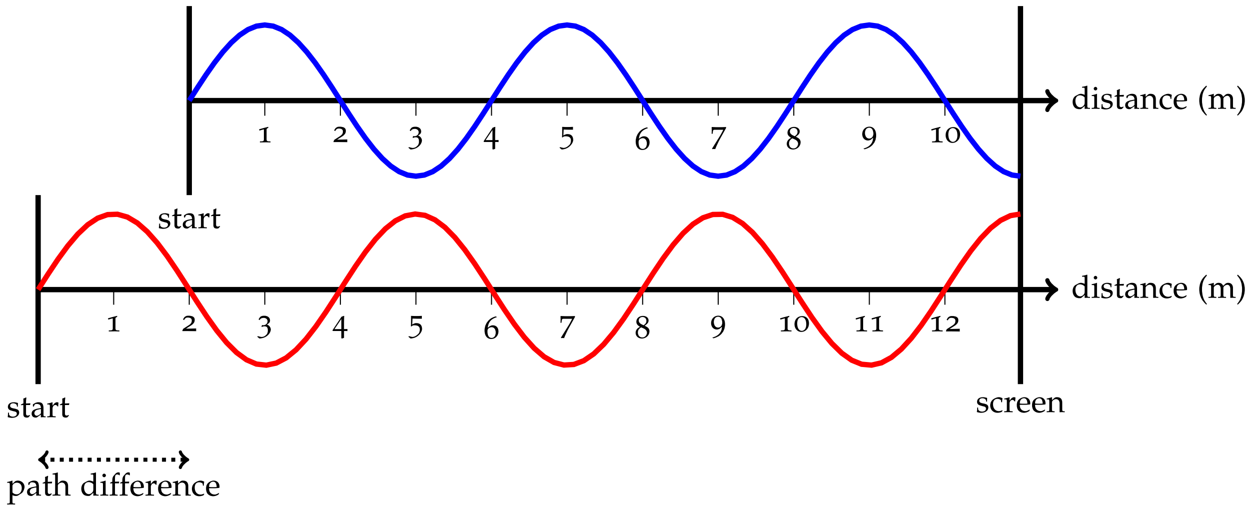

Figure 9: Two Waves Travelling Different Distances

are emitted from their respective start points in phase, and since they travel at the same speed then when they hit the screen they will be out of phase (that is: their phase difference is 2m, or $180°$, or $\frac{\lambda}{2}$, however you want to describe it). Another way of saying that is that the two waves will destructively interfere at the screen, and there will be a dark spot, since the two waves cancel each other out. (See Smith (2015b).)

The *path difference* is defined to be the difference in distances that the two waves travel. In the case of the waves in Figure 9 the path difference is 2 m. So path difference *will be a distance*.

Clearly knowledge of the path difference will be important when figuring out whether two waves will hit a particular point in phase or not.

Now, looking at all the different ways that we can write a phase difference (see Section 1.2), the ones that are distances are:

- the absolute distance (measured in meters, presumably), and
- the relative distance, measured in the number of wavelengths.

And really, the best one to use would be the number of wavelengths. Why?? Because it doesn't really matter what the actual distance involved was; what's important is how many wavelengths the path difference represents. If you knew that the path difference was $\frac{\lambda}{2}$, then you know that the waves will destructively interfere, *whatever their wavelengths are*. You see, the actual distance is only of use when you compare it to the wavelength of the waves.

In Figure 9, for example, the path difference of 2 m is only of use when you compare that to the wavelength of the waves (4 m). So we might as well just use the number of wavelengths in the first place!

OK. Let's have a path difference of λ. See Figure 10. Since the path difference is a whole

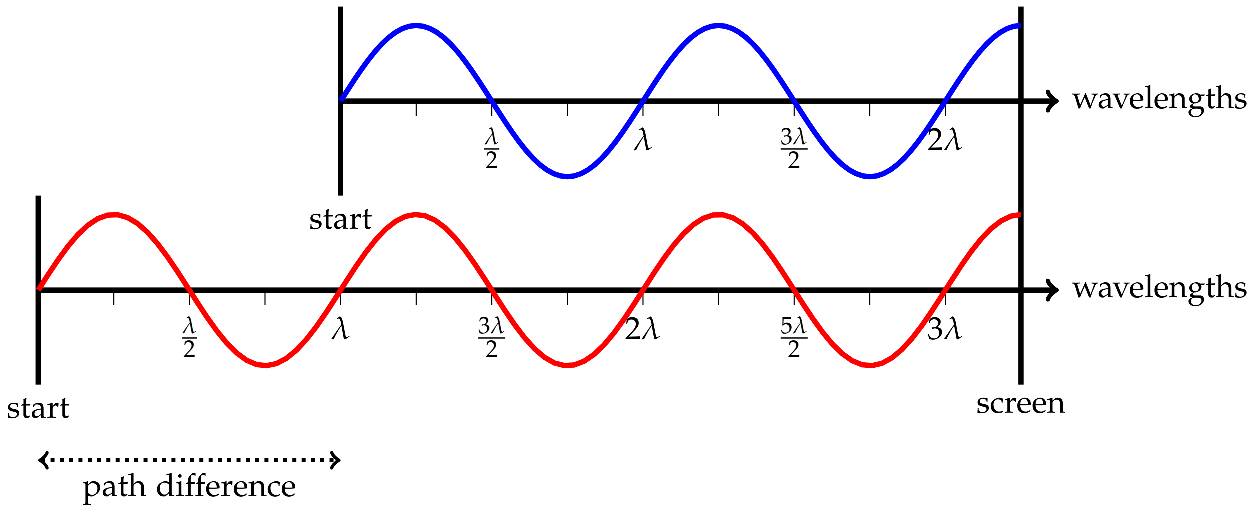

Figure 10: A Path Difference of λ

wavelength, then even though the waves have different distances to travel, they will still hit the screen in phase, so there will be constructive interference, and a bright spot on the screen.

What about when the path difference is $\frac{3\lambda}{2}$?

Or 2λ? Can you see how this is going to work?

2.1 Summary of Path Difference

- Use numbers of wavelengths for the unit of path difference between two waves.
- When the path difference between two waves is λ, or 2λ, or 3λ, or indeed $n\lambda$ (where n is any whole number), then the two waves will be in phase at the target. So constructive interference occurs at the target.
- When the path difference between two waves is a whole number of wavelengths *plus half a wavelength*, that is $\frac{\lambda}{2}$, or $\frac{3\lambda}{2}$ $\left(\lambda + \frac{\lambda}{2}\right)$, or $\frac{5\lambda}{2}$ $\left(2\lambda + \frac{\lambda}{2}\right)$, or indeed $\frac{m\lambda}{2}$ (where m is any *odd* whole number), then the two waves will be out of phase at the target. So destructive interference occurs at the target.

A Phase

Here's the mathematical definition of phase: the *phase* of a wave is actually the angle within a cycle of the wave, at the origin.

If that means nothing to you, let me give you a few examples. The phase of this wave is 90°

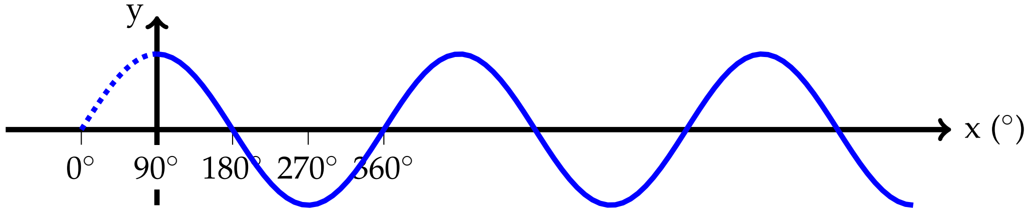

Figure 11: A Wave With a Phase of 90°

because that's what you'd have to shift the sine wave by *to the left* to get this wave. Mathematically, the equation for this wave will be

$$y = \sin(x + 90^\circ)$$

Here's another example. The phase of this wave is 270° because that's what you'd have to shift

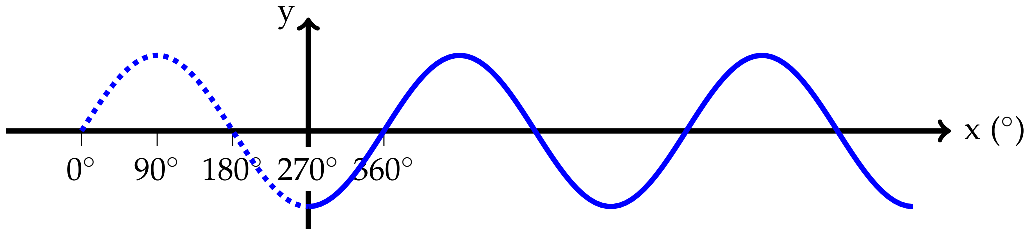

Figure 12: A Wave With a Phase of 270°

the sine wave by *to the left* to get this wave. Mathematically, the equation for this wave will be

$$y = \sin(x + 270^\circ)$$

Here's the final example. The phase of this wave is 135° because that's what you'd have to shift

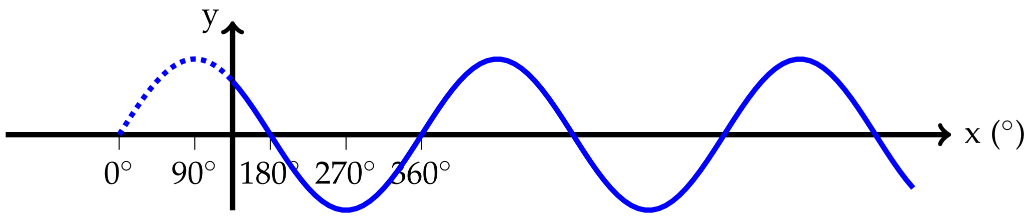

Figure 13: A Wave With a Phase of 135°

the sine wave by *to the left* to get this wave. Mathematically, the equation for this wave will be

$$y = \sin(x + 135^\circ)$$

So in general, the phase (ϕ) of a wave is used like this:

$$y = \sin(x + \phi)$$

References

Smith, S. (2015a). Introduction to Waves. An explanation of the vital statistics of waves.

Smith, S. (2015b). Superposition of Waves. An explanation of the phenomenon of superposition of waves.