Steve Smith Tuition: Physics Notes

Interference and Diffraction of Light Through Slits: The "Just Give Me The Formulae" Version!

Contents

Document History

Date	Version	Comments
14th February 2011	1.0	Initial creation of the document.
30th May 2013	1.1	Updating to clarify and simplify the ideas.

Prerequisites

None.

Notes

None.

1 One Slit

If you shine plane waves of light at a hole in a wall (this is what physicists call a "slit") you get (if you choose the wavelength of the light and the width of the slit appropriately) an interesting pattern of light on a screen placed on the other side of the slit.

You might expect that the light just goes straight through the slit in a straight line, so you would just see a single spot of light on the screen. And actually, most of the time, that's exactly what you do see. But if you choose the width of the slit to be about the same size as the wavelength of the light (which is very small: about $5 \times 10^{-7}m$), then something very interesting happens.

To see what happens, you need to go and visit this web page: Single Slit Diffraction where you will see a Java applet of a single-slit diffraction pattern (see Hwang (2005a)). You should see something like Figure 1.

[Note that in these figures, the sizes of the slits and the distance between slits (see later) are shown much larger than they would appear in practice. As I say, for significant diffraction to occur, the slit width b needs to be around the same size as the wavelength λ of the light. Usually also, the distance between slits is much greater than the individual slit width.] Have a play with the wavelength, slit width and intensity, to get a

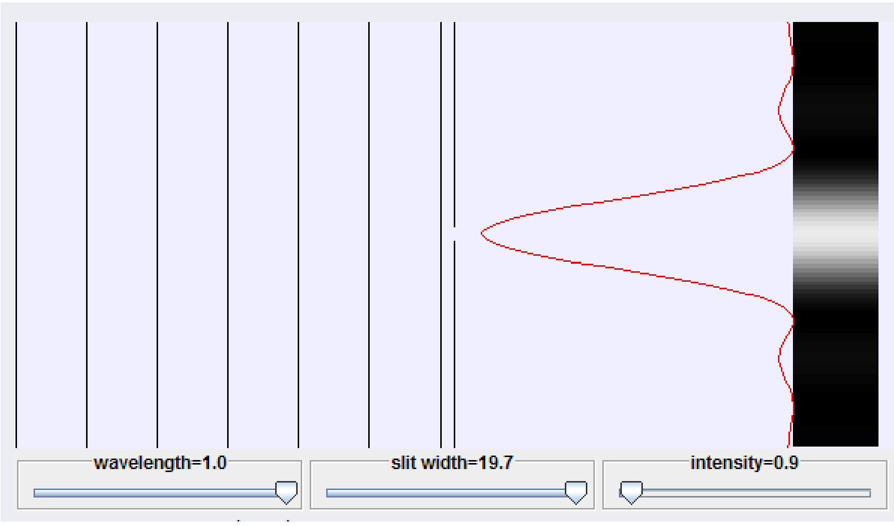


Figure 1: Single Slit Diffraction

bit of a feel for what's going on.

Hopefully, you should be able to see a number of interesting features of the pattern of light on the screen:

- there is a bright area on the screen straight in front of the slit. However, this area is wider than the slit and the size of this area can be changed by changing the relative sizes of the wavelength of the light and the slit width;
- on either side of the central bright area is a region of darkness(!);
- outside those dark areas there is a pair of bright regions, but the brightness of those regions is much less than that of the central bright area;
- continuing out on either side, we have alternating bright and dark areas that seem to go on forever. However, each bright area is much less bright than the one next to it (towards the centre of the screen).

If the angle from the middle of the slit to a point on the screen is θ, and the width of the slit is b, it turns out that for the first **dark** spot either side of the central bright area on the screen (see Figure 2),

$$b \sin(\theta_1) = \lambda$$

Then, for the second **dark** spot,

$$b \sin(\theta_2) = 2\lambda$$

and

$$b \sin(\theta_3) = 3\lambda$$

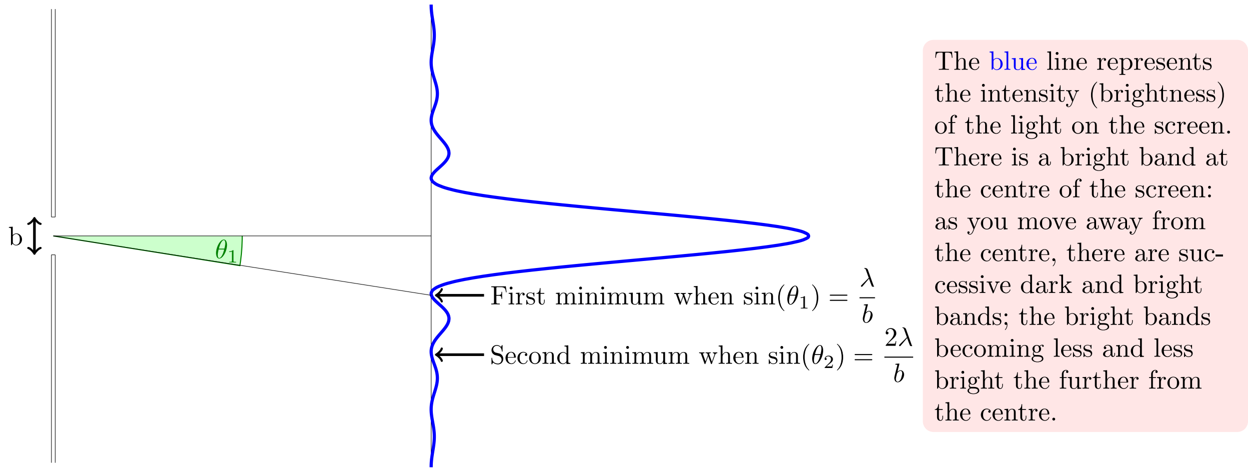


Figure 2: Single Slit Diffraction Formula

for the third, and so on. In general, for the mth **dark** spot from the centre of the screen

$$b\sin(\theta_m) = m\lambda$$

It is interesting to note that the angles where the bright spots appear are NOT half-way between the angles for the dark spots. For example, the bright spot that appears between $m = 1$ and $m = 2$ on the screen is NOT at $m = 1.5$, but is actually at about $m = 1.43$. But for A-Level, you are not expected to know that!

For single slit diffraction,
where b is the slit width,
θ_m the angle to the m^{th} dark spot:

$$\boldsymbol{b\sin(\theta_m) \ = \ m\lambda}$$

1.1 One Slit Screen Distances

Now if you check out Appendix B, we can use the $\sin(\theta)$ approximation in this situation, because θ is small. That means, for example, that the distance on the screen from the centre of the diffraction pattern to the first **dark** spot, x, would be given by:

$$\sin(\theta_1) \approx \frac{x}{D} = \frac{\lambda}{b}$$

or

$$x = \frac{\lambda D}{b}$$

And the width of the central bright region, y, would be

$$y = \frac{2\lambda D}{b}$$

because y is just twice as big as x (see Figure 3).

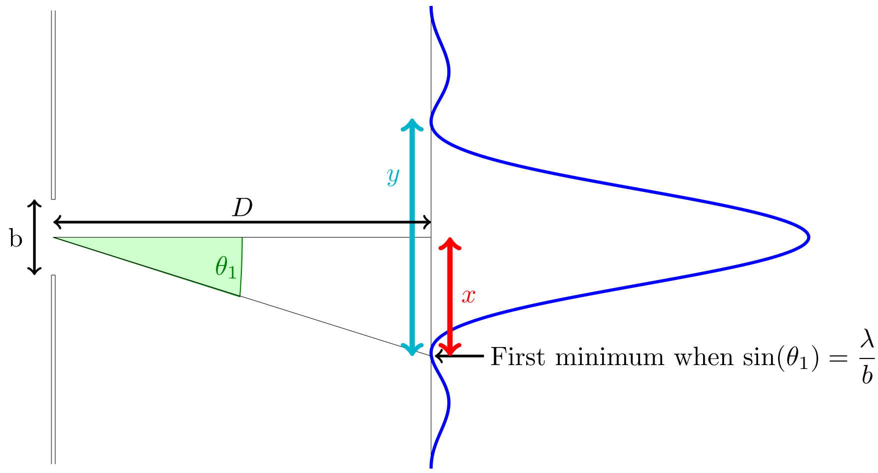


Figure 3: Single Slit Screen Distances

2 Two Slits

OK, so what if you have two slits? If you shine plane waves of light at a pair of holes in a wall (this is what physicists call a "double slit") you get (if you choose the wavelength of the light and the width of the slit appropriately) an interesting pattern of light on a screen placed on the other side of the slits.

You might expect that the light just goes straight through the slits in straight lines, so you would just see a pair of spots of light on the screen. And actually, most of the time, that's exactly what you do see. But if you choose the width of the slits to be about the same size as the wavelength of the light (which is very small: about $5 \times 10^{-7}m$), then something very interesting happens.

To see what happens, you need to go and visit this web page: Multiple Slit Interference where you will see a Java applet of multiple-slit interference patterns (see Hwang (2005b)). You should see something like Figure 4. Have a play with the various parameters you can change, to get a bit of a feel for what's going on.

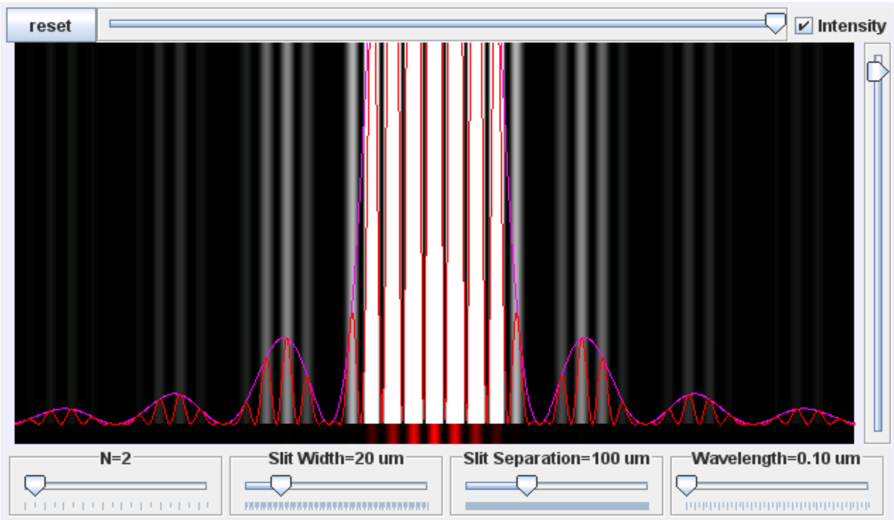
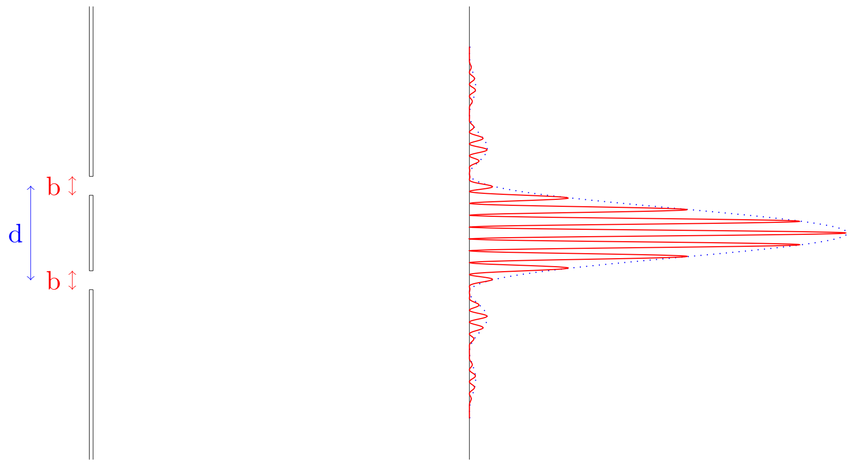


Figure 4: Multiple Slit Interference

The pattern of light on the screen now consists of two parts: the one-slit pattern that we saw in section 1; and within that a new pattern that has many more peaks and troughs in it. Exactly how many peaks there will be depends on the values of the three variables that you can modify here:

- the wavelength of the light, λ;
- the width of the slits, b;
- the distance between the slits, d.



The blue line represents the intensity (brightness) of the light on the screen. There is a bright band at the centre of the screen: as you move away from the centre, there are successive dark and bright bands; the bright bands becoming less and less bright the further from the centre.

Figure 5: Double Slit Interference 1

I've used Hwang's Multiple Slit Interference program to help draw Figures 5 and 6. These patterns were obtained when the variables in the program have the values $\lambda = 1\mu$m, $b = 20\mu$m and $d = 100\mu$m. [Figure 6 is just the same picture as Figure 5 but looking at a smaller part of the screen. This is so that we can see where the angles θ_1 and θ_2 come from.]

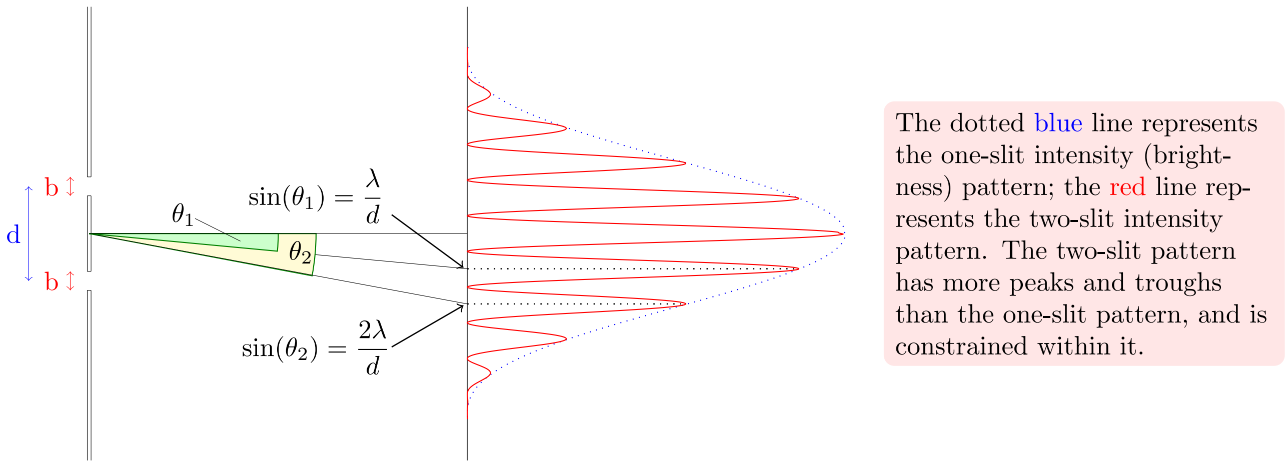


Figure 6: Double Slit Interference 2

If the angle from the middle of the slits to a point on the screen is θ, and the distance between the slits is d, it turns out that for the first **bright** spot either side of the central bright area on the screen (see Figure 6), then

$$d \sin(\theta_1) = \lambda$$

This is called the *first order* diffraction image. Then, for the second **bright** spot (the *second order* diffraction image),

$$d \sin(\theta_2) = 2\lambda$$

and

$$d \sin(\theta_3) = 3\lambda$$

for the third order diffraction image, and so on. In general, for the m^{th} **bright** spot from the centre of the screen

For double slit diffraction,
where d is the slit separation,
θ_m the angle to the m^{th} bright spot:

$$d \sin(\theta_m) = m\lambda$$

Notice how similar the equations are for the angles to the **bright** parts of the screen with two slits to the equations for the angles to the **dark** parts of the screen with one slit. This is very confusing!

2.1 Two Slit Screen Distances

Now if you check out Appendix B, we can use the $\sin(\theta)$ approximation in this situation, because θ is small. That means, for example, that the distance on the screen from the centre of the diffraction pattern to the first **bright** spot, x, would be given by:

$$\sin(\theta_1) \approx \frac{x}{D} = \frac{\lambda}{d}$$

or

$$x = \frac{\lambda D}{d}$$

And the distance on the screen from the centre of the diffraction pattern to the second **bright** spot, y, would be given by:

$$\sin(\theta_2) \approx \frac{y}{D} = \frac{2\lambda}{d}$$

or

$$y = \frac{2\lambda D}{d}$$

(see Figure 7).

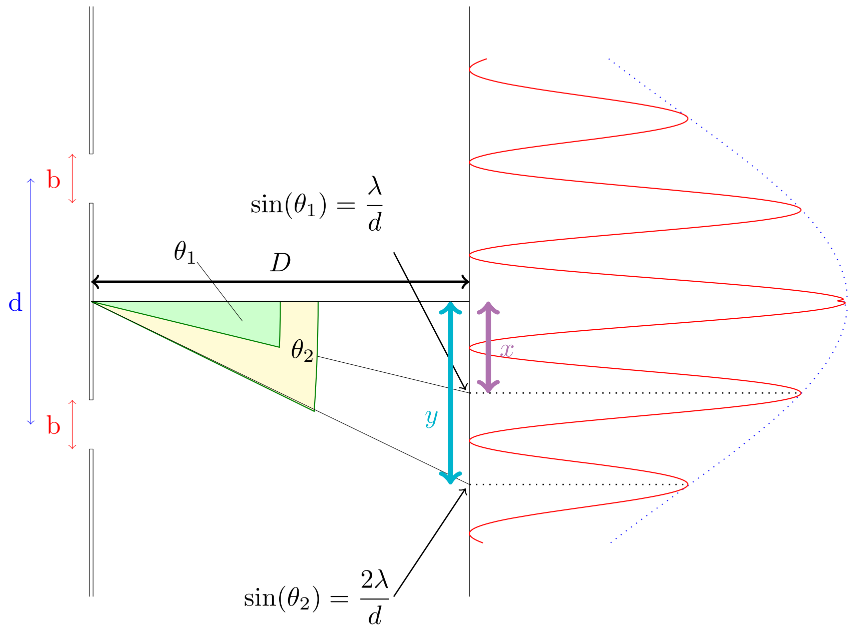


Figure 7: Double Slit Screen Distances

2.2 Missing Orders

You can see from the figures in this section that the red two-slit diffraction pattern is always constrained within the blue one-slit diffraction pattern. Also, the formula for the two-slit pattern tells us where **bright** spots in that pattern are, whilst the formula for the one-slit pattern tells us where the **dark** spots in that pattern are.

So what happens if a bright spot in the two-slit pattern coincides with a dark spot in the one-slit pattern?

Well, as you can see from the figures, the one-slit pattern predominates. So if a bright spot in the two-slit pattern coincides with a dark spot in the one-slit pattern, then you just wouldn't see that bright spot. It would be called a *missing order*.

Figure 8 is the same picture as Figure 5, except that it has been expanded a bit, and the region of the screen where there is a missing order has been magnified. It is the 5^{th} order maximum in the two-slit pattern that is missing.

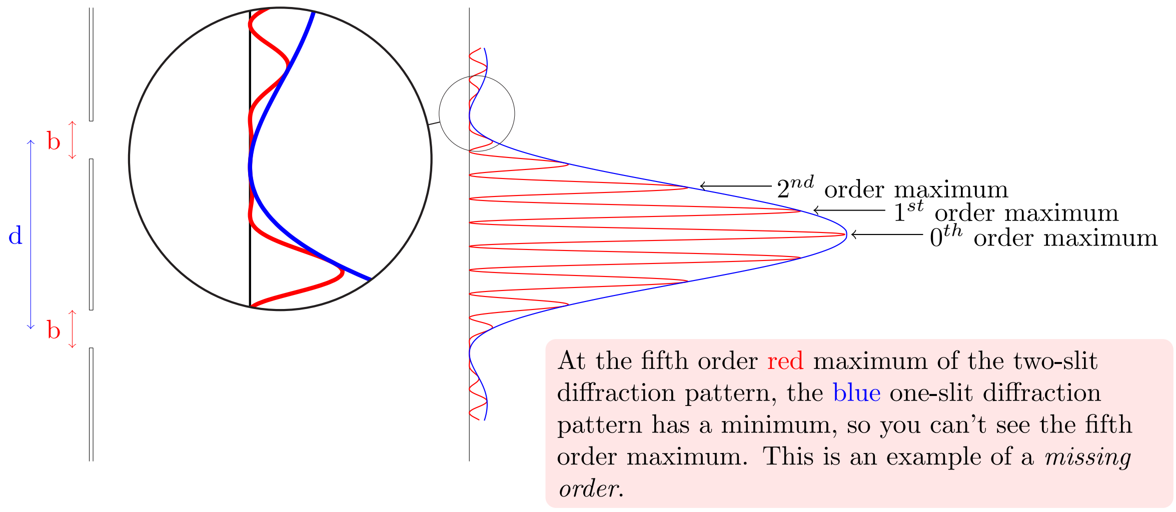


Figure 8: Two Slits: Missing Orders

Now from what we've got so far, the distance on the screen from the centre of the diffraction pattern to the first dark spot is

$$x = \frac{\lambda D}{b}$$

and the two-slit maxima occur at distances of

$$x = \frac{m\lambda D}{d}$$

So, if one of our two-slit maxima occurs at the one-slit dark spot then

$$\frac{\lambda D}{b} = \frac{m\lambda D}{d}$$

and so

$$m = \frac{d}{b}$$

And for the Figures in this section, remember that I said that they had been drawn with values of $b = 20\mu$m and $d = 100\mu$m? So that's why the first missing order is the 5^{th}! Because $\frac{d}{b} = 5$! It all hangs together!

3 Many Slits

When you have five slits the picture looks like that in Figure 9. As the number of slits increases, certain peaks in the red pattern become prominent, and other peaks seem to be cancelled out. And the more slits you have, the more prominent these certain peaks become, and the rest seem to dwindle away to nothing.

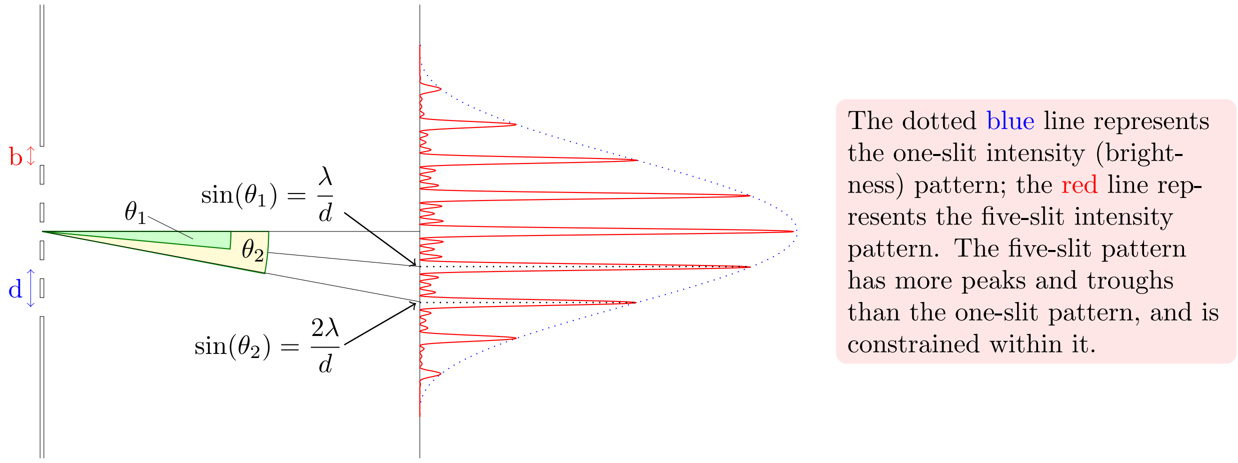


Figure 9: Multiple Slit Interference

And all the time, no matter how many slits you have, all these red peaks never escape the blue one-slit diffraction pattern.

And just like the case of two slits, it turns out that the **bright** spots on the screen obey the relationship:

For multi-slit diffraction,
where d is the slit separation,
θ_m the angle to the m^{th} bright spot:

$$d\sin(\theta_m) = m\lambda$$

3.1 The Diffraction Grating

A diffraction grating is just a thing with loads of slits in it. I mean, hundreds of thousands of slits. All evenly spaced. But is doesn't matter how many slits there are, the equation that applies is just that in section 3. The d in the equation is just the distance between any pair of adjacent slits.

The only real difference between having essentially an infinite number of slits as we have in a diffraction grating, and two slits, is that because of all those maxima that shrivel to nothing in the many-slit case, the maxima that are left are all very narrow.

4 Diffraction and Interference

Diffraction is the name given to the phenomenon where waves spread out as they go through gaps, or bend around corners of objects. Interference is just the superposition of waves (see Smith (2011b)). So what links the two? Well: Diffraction IS Interference! Or rather, diffraction is caused by interference. Because diffraction is caused by the superposition of many waves during the normal progression of any wave. To see how this happens, you'll have to go and read Smith (2011a).

5 Summary

For single slit diffraction,
where b is the slit width,
θ_m the angle to the m^{th} dark spot:

$$b\sin(\theta_m) = m\lambda$$

For multi-slit diffraction,
where d is the slit separation,
θ_m the angle to the m^{th} bright spot:

$$d\sin(\theta_m) = m\lambda$$

For small angles of θ,

$$\sin(\theta) \approx \frac{x}{D}$$

where x is the distance on the screen
and D is the slits-screen distance

A $\sin(\theta)$ and $\tan(\theta)$ for Small Angles θ

Have a look at the values of $\sin(\theta)$ and $\tan(\theta)$ for some small angles of θ in Table 1:

Table 1: Values of $\sin(\theta)$ and $\tan(\theta)$ for Small Angles θ

$\theta(^\circ)$	$\sin(\theta)$	$\tan(\theta)$	% Error
1	0.0175	0.0175	0.0
2	0.0349	0.0349	0.1
3	0.0523	0.0524	0.1
4	0.0698	0.0699	0.2
5	0.0872	0.0875	0.4
10	0.1736	0.1763	1.5
15	0.2588	0.2679	3.5
20	0.3420	0.3640	6.4

Table 1 shows that if the angle θ is small, $\sin(\theta)$ has a very similar value to $\tan(\theta)$. And the smaller the θ, the closer the two values are.

So, if you are faced with a situation where your angles θ are small, then

$$\sin(\theta) \approx \tan(\theta) \tag{1}$$

and wherever you have a $\sin(\theta)$, you could replace it with $\tan(\theta)$, and vice versa.

For small angles of $\boldsymbol{\theta}$,

$$\boldsymbol{\sin(\theta) \approx \tan(\theta)}$$

B The Approximation For $\sin(\theta)$

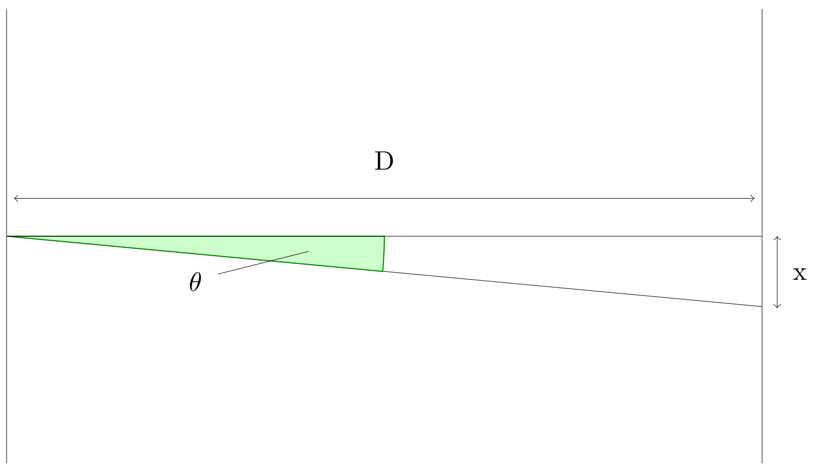


Figure 10: The approximation for $\sin(\theta)$

Now, check out Figure 10. From basic trigonometry,

$$\tan\theta \;=\; \frac{x}{D}$$

But, if the angle θ is small, then $\sin(\theta)$ will be approximately the same as $\tan(\theta)$ (see Appendix A). That means that

$$\sin(\theta) \approx \frac{x}{D}$$

So if you are carrying out a diffraction experiment, and you are trying to measure a distance on the screen (x), then you can relate that distance to the slits-to-screen distance D and the sine of the angle θ by:

For small angles of $\boldsymbol{\theta}$,

$$\sin(\theta) \;\approx\; \frac{x}{D}$$

References

Hwang, F.-K. (2005a). Single Slit Diffraction. Website. http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=128.0.

Hwang, F.-K. (2005b). Single/Multiple Slit Diffraction/Interference. Website. http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=129.0.

Smith, S. (2011a). Diffraction II. Explains how diffraction is caused by superposition of waves.

Smith, S. (2011b). Superposition of Waves. Explanation of the phenomenon of superposition of waves.