Steve Smith Tuition: Maths Notes

AQA Astrophysics Option Lenses Summary Questions

Contents

Prerequisites

None.

Notes

None.

Document History

Date	Version	Comments
30 March 2012	1.0	Initial creation of the document.

1 Lenses: Question 1

1.1 The Question

(a)(i) Copy and complete the ray diagram to show how a converging lens in a camera forms an image of an object.

(a)(ii) State whether the image is real or virtual, magnified or diminished, upright or inverted.

(b)(i) Draw a ray diagram to show how a converging lens is used as a magnifying glass.

(b)(ii) State whether the image is real or virtual, magnified or diminished, upright or inverted.

1.2 The Answer

(a)(i) This question is about the use of the lens *in a camera*. In a camera the object is more than two focal lengths from the lens.

So, the first thing we do is to draw the principal axis (the horizontal line through the middle if the lens), the lens, the focal points F and $2F$ either side of the lens (this gets to be standard practice), and the object (see Figure 1).

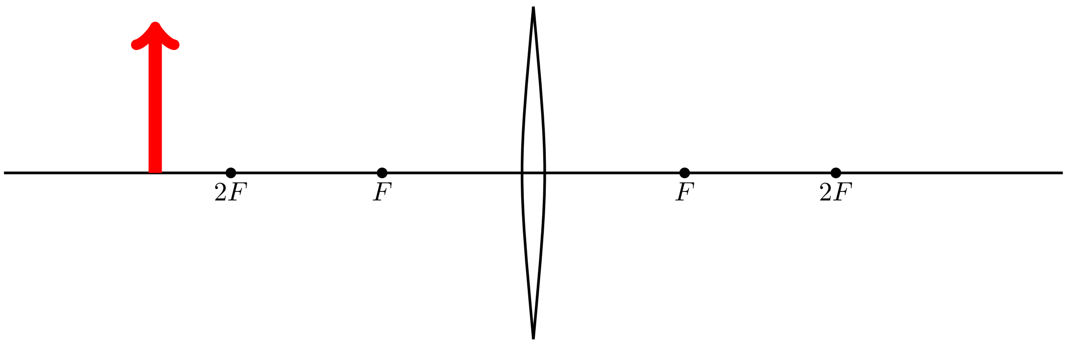

Figure 1: Lens, axis, focal points and object

The next thing we do is to draw in a ray from the top of the object parallel to the lens axis, to the lens and out the other side. The reason we choose a ray parallel to the axis is because we know that such a ray will pass through the focal point (F) on the other side of the lens. The lens is designed to do just that (see Figure 2).

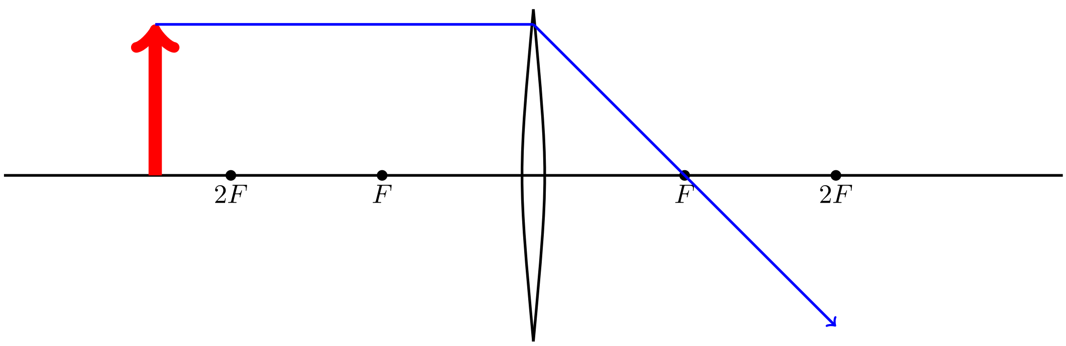

Figure 2: Addition of a hoizontal ray from the top of the object

Now we add the ray from the top of the object through the centre of the lens. Lenses are designed so that such a ray will pass straight through the lens without being refracted (see Figure 3).

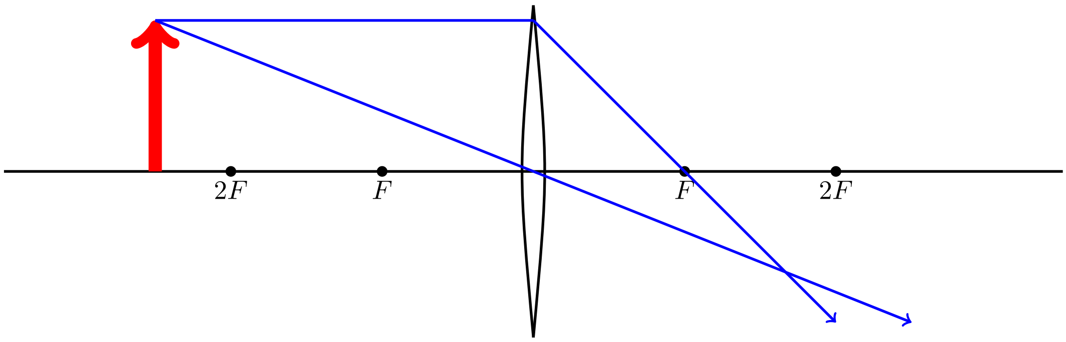

Figure 3: Addition of a ray through the centre of the lens

Now where these rays cross will be the location of the image of the top of the object. Since the image of the bottom of the object must lie on the principal axis (why??), we can now draw the image in (see Figure 4).

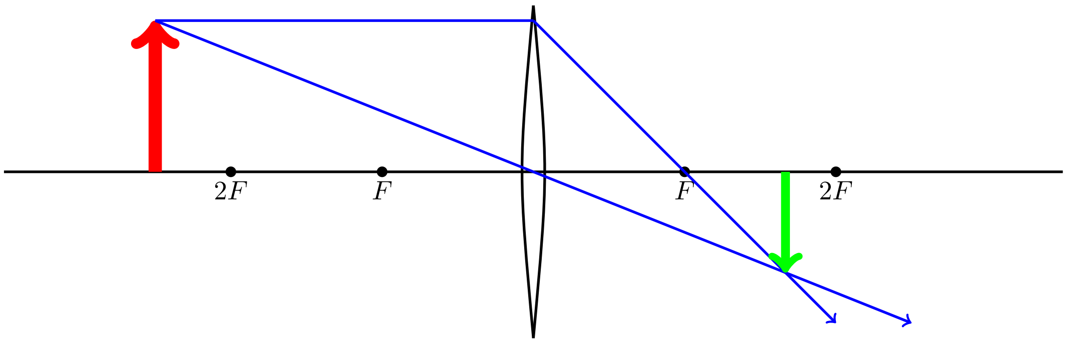

Figure 4: Location of the image

In your course, it is suggested that you draw three rays: the two we have already drawn, and a third one that goes through the focal point on the same side of the lens as the object (see Figure 5). This ray will

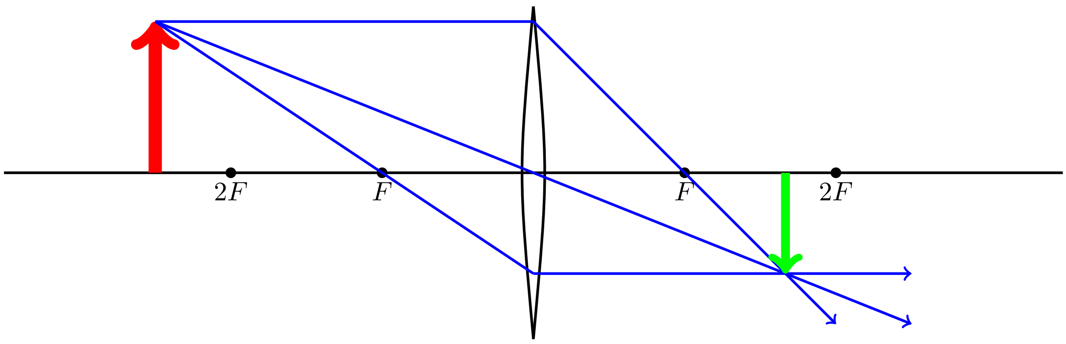

Figure 5: Third ray

emerge parallel to the axis on the other side of the lens (why??). Notice that you don't really need to draw this ray in. It's really only needed for confirmation of the location of the top of the image.

(a)(ii) This image will be real, diminished and inverted. You can see from the diagram that the image will be diminished and inverted.

To see why it is real, you could use the lens formula. In Figures 1 to 5, I have drawn the image at -5 cm on the x-axis, and I have chosen my focal length to be 2 cm (see Figure 6). So, from the lens formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

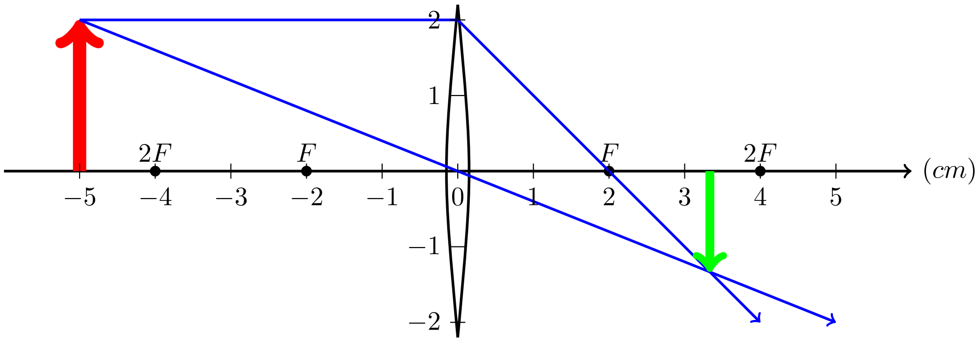

Figure 6: Calculations...

then in our case, $u = 5\ cm$ and $f = 2\ cm$. So:

$$\frac{1}{5} + \frac{1}{v} = \frac{1}{2}$$

or

$$\frac{1}{v} = \frac{1}{2} - \frac{1}{5} = \frac{5}{10} - \frac{2}{10} = \frac{3}{10}$$

so

$$v = \frac{10}{3}\ cm$$

Hence the image location is at distance of $\frac{10}{3}$ cm from the lens. Since v is positive, the image is real (remember: *real is positive, virtual is negative*). You can also tell that it's real in practice because you could put a screen (a piece of paper, say) where the image is and you would see the image on the screen.

And we could calculate the magnification of the image in this position:

$$magnification = \frac{v}{u} = \frac{\frac{10}{3}}{5} = \frac{2}{3}$$

so indeed, the image is diminished.

Notice that to see the image through the lens we would have to look through the lens from the left.

(b)(i) When the lens is used as a magnifying glass, the object needs to be between the lens and the focal point (see Figure 7).

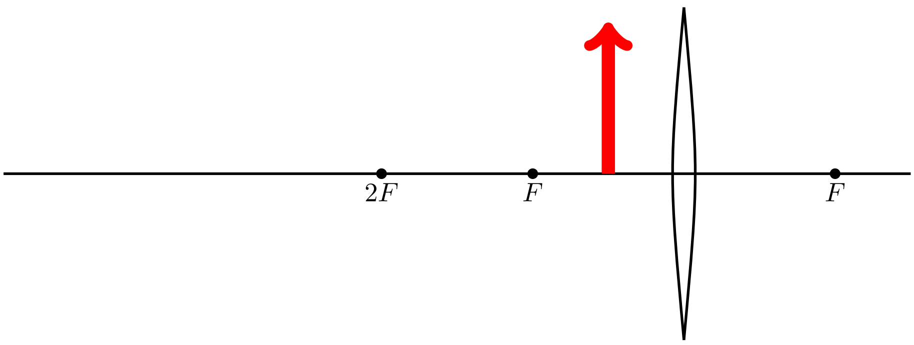

Figure 7: Lens, axis, focal points and object

We now go through the same process as before, drawing in the parallel ray (see Figure 8).

And because I know what's coming (!) I have drawn this ray going both ways from the lens. The solid line denotes the actual ray path. The dotted line represents the direction an observer on the right of the lens thinks the ray is coming from.

Now we draw in the ray that goes through the centre of the lens (see Figure 9), again using a solid line to

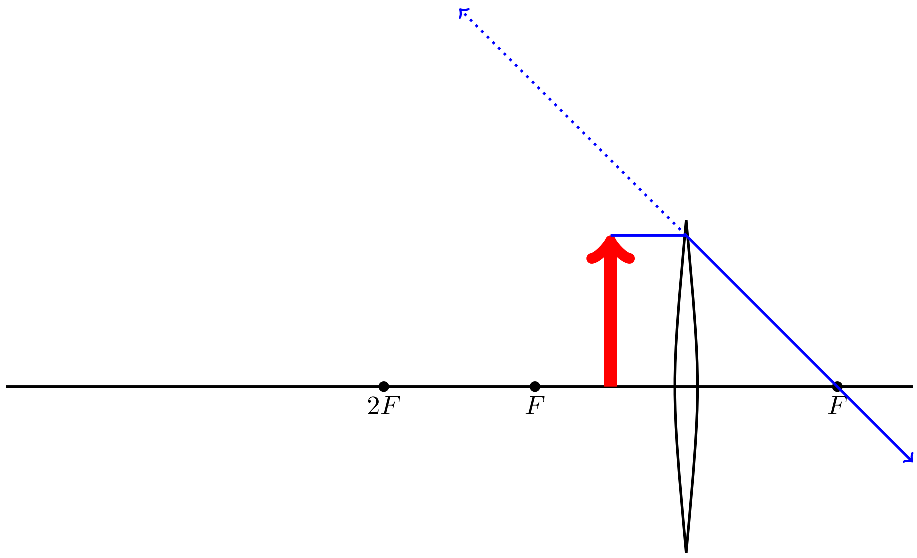

Figure 8: Adding ray parallel to the axis

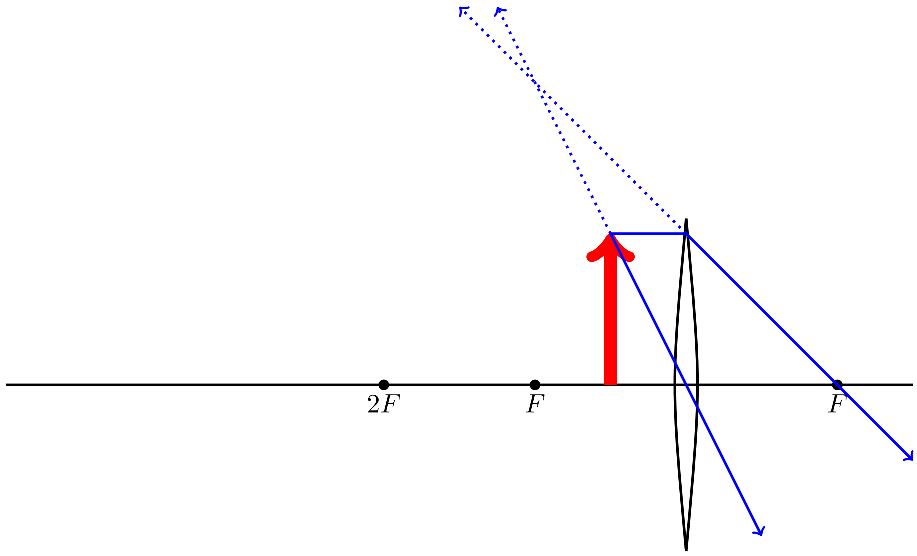

Figure 9: Adding ray through the centre of the lens

represent the actual ray, and a dotted line to represent the direction that the observer on the right of the lens would think the ray was coming from.

The place where the lines cross will again be the location of the top of the image. So we can now draw this in (see Figure 10).

(b)(ii) This image will be virtual, enlarged and upright. You can see from Figure 10 that the image will be enlarged and upright.

To see why it is virtual, you could use the lens formula. In my diagrams, I have drawn the image at -1 on the x-axis, and again I have chosen my focal length to be 2. So, from the lens formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

then in our case, $u = 1$ and $f = 2$. So:

$$\frac{1}{1} + \frac{1}{v} = \frac{1}{2}$$

or

$$\frac{1}{v} = \frac{1}{2} - \frac{1}{1} = \frac{1}{2} - \frac{2}{2} = -\frac{1}{2}$$

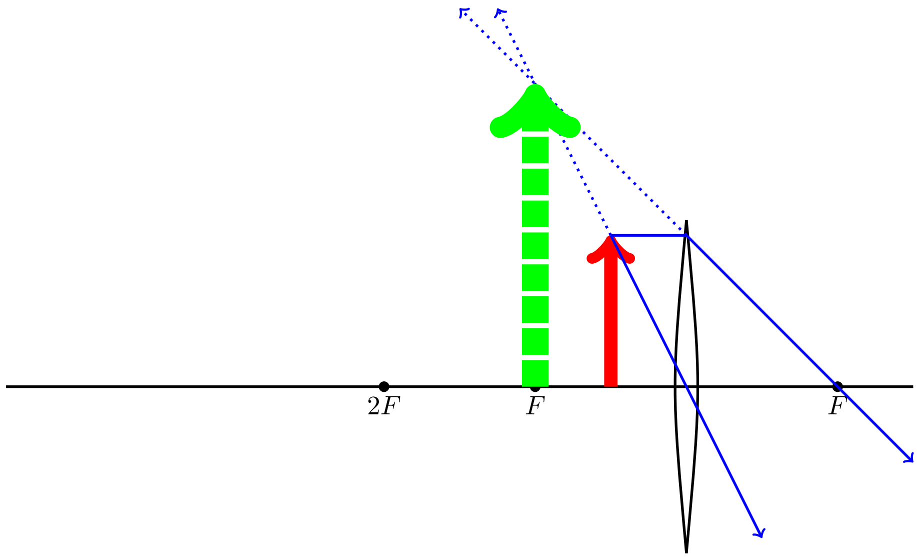

Figure 10: Location of the image

so

$$v = -2$$

Hence the image location is at a distance of 2 from the lens. [The sign of the v determines whether the image is real or virtual; the value of v determines the image distance from the lens.] Since v is negative, the image is virtual (remember: *real is positive, virtual is negative*).

And we could calculate the magnification of the image in this position:

$$magnification = \frac{v}{u} = \frac{2}{1} = 2$$

so indeed, the image is enlarged. [Note that in calculating the magnification, we are only interested in the values of u and v, not their signs. So in calculating magnification, we only use positive values for u and v.]

Notice that to see the image through the lens we would have to look through the lens from the right.

2 Lenses: Question 2

2.1 The Question

An object is placed on the principal axis of a thin converging lens at a distance of 400 mm from the centre of the lens. The lens has a focal length of 150 mm.

(a) Draw a ray diagram to determine the distance from the image to the lens.

(b) State whether the image is (i) real or virtual, (ii) upright or inverted.

(c) Use the lens formula to check the accuracy of your ray diagram.

2.2 The Answer

(a) The diagram for this situation is shown in Figure 11.

(b)(i)(ii) This image will be real, diminished and inverted. You can see from the diagram that the image will be diminished and inverted.

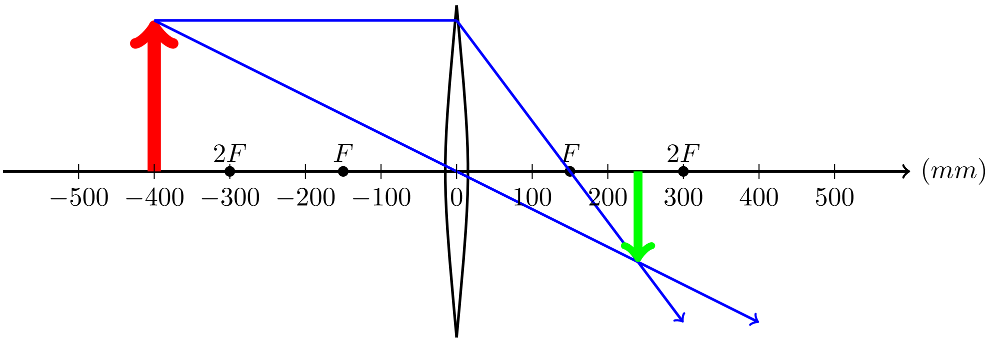

Figure 11: Diagram for Question 2

(c) To see why it is real, and to calculate precise values for image distance and magnification, you could use the lens formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

then in our case, $u = 400\ mm$ and $f = 150\ mm$. So:

$$\frac{1}{400} + \frac{1}{v} = \frac{1}{150}$$

or

$$\frac{1}{v} = \frac{1}{150} - \frac{1}{400} = \frac{8}{1200} - \frac{3}{1200} = \frac{5}{1200}$$

so

$$v = \frac{1200}{5} = 240\ mm$$

Hence the image location is at a distance of 240 mm from the lens. Since v is positive, the image is real (remember: *real is positive, virtual is negative*).

And we could calculate the magnification of the image in this position:

$$magnification = \frac{v}{u} = \frac{240}{400} = \frac{3}{5}$$

so indeed, the image is diminished.

Notice that to see the image through the lens we would have to look through the lens from the left.

3 Lenses: Question 3

3.1 The Question

An object is placed on the principal axis of a thin converging lens at a distance of 100 mm from the centre of the lens. The lens has a focal length of 150 mm.

(a) Draw a ray diagram to determine the distance from the image to the lens.

(b) State whether the image is (i) real or virtual, (ii) upright or inverted.

(c) Use the lens formula to check the accuracy of your ray diagram.

3.2 The Answer

(a) The diagram for this situation is shown in Figure 12.

(b)(i)(ii) This image will be virtual, enlarged and upright. You can see from the diagram that the image will be enlarged and upright.

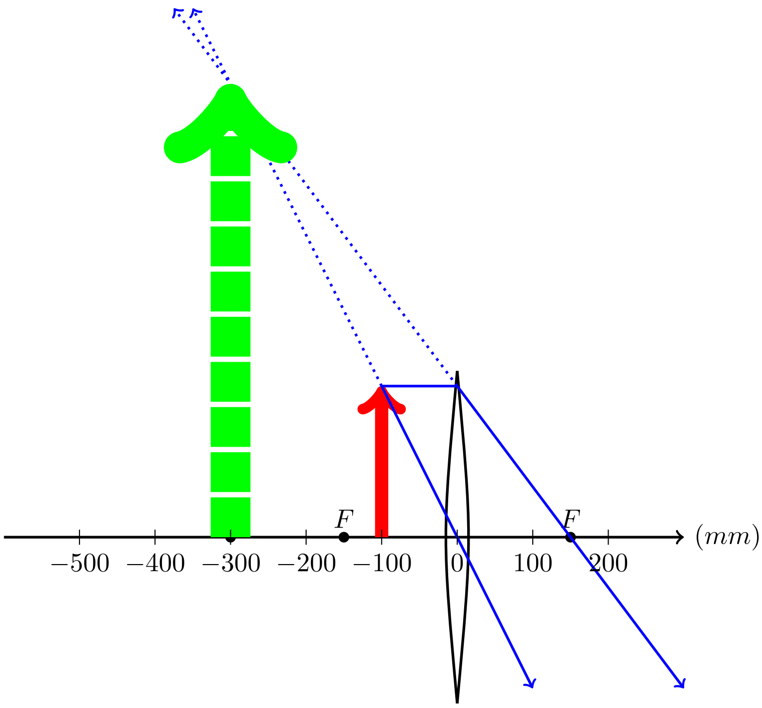

Figure 12: Diagram for Question 3

(c) To see why it is virtual, and to calculate precise values for image distance and magnification, you could use the lens formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

then in our case, $u = 100\ mm$ and $f = 150\ mm$. So:

$$\frac{1}{100} + \frac{1}{v} = \frac{1}{150}$$

or

$$\frac{1}{v} = \frac{1}{150} - \frac{1}{100} = \frac{2}{300} - \frac{3}{300} = -\frac{1}{300}$$

so

$$v = -300\ mm$$

Hence the image location is at a distance of 300 mm from the lens. Since v is negative, the image is virtual (remember: *real is positive, virtual is negative*).

And we could calculate the magnification of the image in this position:

$$magnification\ = \frac{v}{u} = \frac{300}{100} = 3$$

so indeed, the image is enlarged.

Notice that to see the image through the lens we would have to look through the lens from the right.

4 Lenses: Question 4

4.1 The Question

An object of height 10 mm is placed on the principal axis of a converging lens of focal length 0.200 m.

(a) Calculate the image distance and the height of the image for an object distance of (i) 0.150 m, (ii) 0.250 m.

(b) In each case, calculate the distance between the object and the image and state whether the image is real or virtual and upright or inverted.

4.2 The Answer

(a)(i) The diagram for this situation is shown in Figure 13.

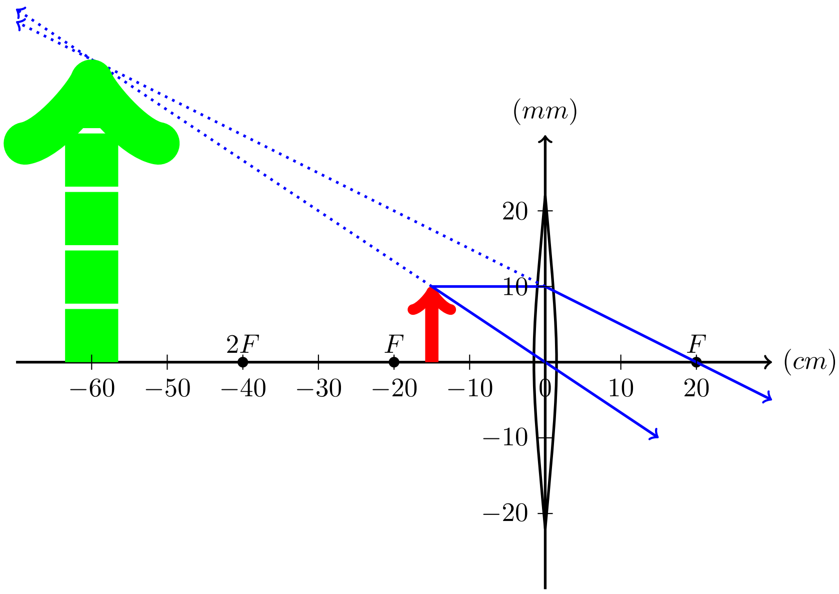

Figure 13: Diagram for Question 4a(i)

(b)(i) This image will be virtual, enlarged and upright. You can see from the diagram that the image will be enlarged and upright.

(c)(i) To see why it is virtual, and to calculate precise values for image distance and magnification, you could use the lens formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

then in our case, $u = 15\ cm$ and $f = 20\ cm$. So:

$$\frac{1}{15} + \frac{1}{v} = \frac{1}{20}$$

or

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{15} = \frac{3}{60} - \frac{4}{60} = -\frac{1}{60}$$

so

$$v = -60\ cm$$

Hence the image location is at a distance of 60 cm from the lens. Since v is negative, the image is virtual (remember: *real is positive, virtual is negative*).

And we could calculate the magnification of the image in this position:

$$magnification = \frac{v}{u} = \frac{60}{15} = 4$$

so indeed, the image is enlarged.

Notice that to see the image through the lens we would have to look through the lens from the right.

(a)(ii) The diagram for this situation is shown in Figure 14.

(b)(ii) This image will be real, enlarged and inverted. You can see from the diagram that the image will be enlarged and inverted.

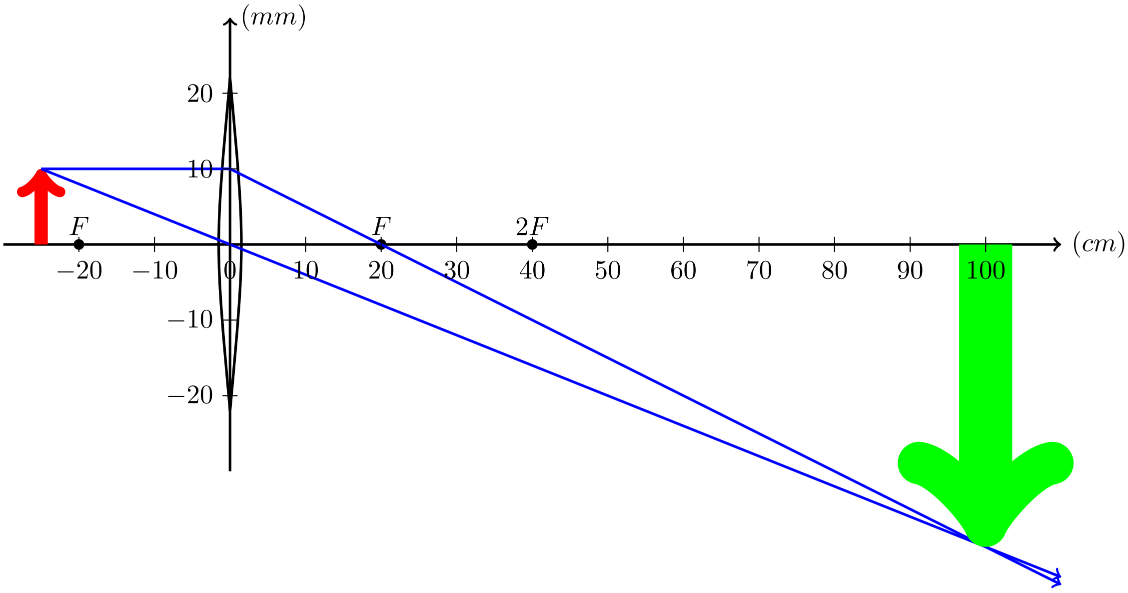

Figure 14: Diagram for Question 4a(ii)

(c)(ii) To see why it is real, and to calculate precise values for image distance and magnification, you could use the lens formula:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

then in our case, $u = 25\ cm$ and $f = 20\ cm$. So:

$$\frac{1}{25} + \frac{1}{v} = \frac{1}{20}$$

or

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{25} = \frac{5}{100} - \frac{4}{100} = -\frac{1}{100}$$

so

$$v = 100\ cm$$

Hence the image location is at a distance of 100 cm from the lens. Since v is positive, the image is real (remember: *real is positive, virtual is negative*).

And we could calculate the magnification of the image in this position:

$$magnification\ = \frac{v}{u} = \frac{100}{25} = 4$$

so indeed, the image is enlarged.

Notice that to see the image through the lens we would have to look through the lens from the left.