Interesting Maths and Physics Questions

Steve Smith

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Question 1

You have a 340 ml bottle of San Miguel beer (5% ABV - alcohol by volume) and a bottle of Jose Cuervo Gold Tequila (40% ABV). How much Tequila would you have to add to the beer to make its strength 6% ABV?

Question 2

In a country where it is illegal to drive a camel if you have had more than 2 units of alcohol [1 unit $\equiv$ 10 ml of pure alcohol], Mohammed is making a Margherita. The recipe Mohammed is following requires 2 parts Tequila (40% ABV), to 1 part Cointreau (also 40% ABV) to 1 part lime juice. What volume of Margherita can he drink to ensure that he is not over the camel drink-drive limit?

Question 3

The relativistic doppler effect is given by

$$\frac{\lambda_o}{\lambda_s} = \sqrt{\frac{1+\beta}{1-\beta}}$$

where:

- λ_o is the observed wavelength of light;
- λ_s is the wavelength actually emitted by the light source;
- $\beta = \frac{v}{c}$
- v is the speed of the observer as it approaches the light source;
- c is the speed of light.

So: how fast would you have to approach a red traffic light for it to look green?
[Wavelength of red light: 650 nm; wavelength of green light: 510 nm]

Question 4

Newton's Gravitation equation is:

$$F(r) = \frac{GMm}{r^2}$$

where:

- G is the universal gravitational constant;
- M is the large mass causing the gravitational field;
- m is the small mass in the field...
- ... (whose center is) a distance r away from (the center of) the large mass M;
- F is the force on the mass m due to the mass M.

If you integrate this function with respect to distance, i.e.

$$\int F.dr$$

then you get the potential energy (E_p) function:

$$E_p = \int \frac{GMm}{r^2} dr$$

so that the potential energy gained by a mass m in moving from a distance r_M from the center of the mass M to a distance $r_M + h$ will be

$$E_p = \int_{r=r_M}^{r=r_M+h} \frac{GMm}{r^2} dr$$

(a)

Treating G, M and m as constants, show that the potential energy gained by a mass m being raised through a small height h from the surface of the mass M (which has a radius r_M) is

$$E_p = GMm \left\{ \frac{h}{r_M(r_M + h)} \right\}$$

(b)

Show further that if $h \ll r_M$ then this can be approximated to

$$E_p = mgh$$

Question 5

The diagram below shows the layout of some streets (that run West-East) and avenues (that run North-South) in New York.

When you arrive at your hotel (PC), Newton Faulkner rings you up. He says that he will have dinner with you that evening if you can tell him, when you arrive at his hotel (NF), how many different ways there are of getting from your hotel to his.

The only rule that you have to obey is that on the streets you can only travel East; and on the avenues you can only travel South.

Question 6

A rather large red mass with a white beard falls off a roof 25 m high on the planet *Coincidence*. Oddly, only the day before, a scientist fell from the same roof while trying to measure the local acceleration due to gravity. Because of her untimely plunge, the scientist was only able to measure this value to within 20% of its actual value. She measured it to be 5 ms^{-2}.

In order to guarantee saving the large red mass before it hits the ground, how long does a nearby reindeer have to act?

Question 7

The perfect martini should be served at a temperature of 38° F[1].

(a)

Determine the ideal martini temperature in degrees Celcius.

[Hint: The conversion formula between Farenheit and Celcius is

$$F = \frac{9}{5}C + 32$$

and this can be used to help find the answer to this part.]

(b)

Tarquin rustles himself up a martini after a hard day watching his thoroughbreds race. The martini has a mass of 250 g. The ingredients for the martini had all been stored at room temperature (around 22° C) prior to the manufacture of the drink.

How much ice needs to be added to the martini to bring the temperature of the cocktail to ideal?

[Useful data:

specific heat capacity of martini ingredients: 3500 $Jkg^{-1}K^{-1}$;
specific heat capacity of water: 4200 $Jkg^{-1}K^{-1}$;
specific latent heat of fusion of ice: 334000 Jkg^{-1}.]

(c)

State any assumptions that you have made in the above calculations.

Question 8

I like going to Fish 'N' Chick'N.

On Monday I ordered one large cod, one regular cod and one large chips. It came to £11.40.

On Tuesday I ordered one regular cod and one regular chips. It came to £5.55.

On Wednesday I ordered one medium cod, one large cod and one regular chips. It came to £11.15.

On Thursday I ordered (one medium cod and one large chips) twice. (I made a new friend in the JobCenter on Wednesday.) It came to £13.60.

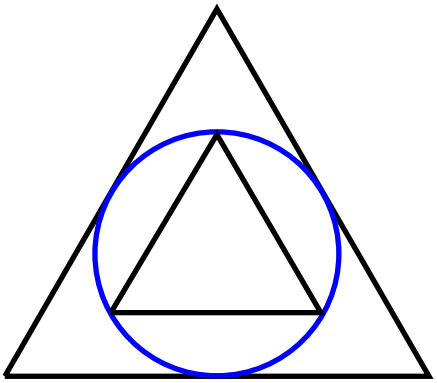
On Friday I ordered three regular cod and one large chips. It came to £14.20.

On Saturday I will be going to Fish 'N' Chick'N. I have in mind to buy one large cod and one large chips. But I only have £7.50 left in my wallet. Will I be able to afford it?

[1] By a strange coincidence, 38° is also the the critical angle at which the incidence of a human-triggered avalanche is greatest.

Question 9

This diagram shows a circle with one equilateral triangle inside and one equilateral triangle outside.



(a)

Calculate the *exact* ratio of the areas of the two triangles.

(b)

Draw a second circle inscribed inside the small triangle. Find the *exact* ratio of the areas of the two circles.